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Total Vertex Irregularity Strength of Hayat Tree Graph

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Abstract
Let $\boldsymbol{G(V,E)}$ be a finite, simple graph with no loop and parallel edges. $\boldsymbol{V}$ is a set of vertices in $\boldsymbol{G}$ and $\boldsymbol{E}$ is a set edges. Labeling a graf is mapping that sends some set of graph elements to a set of positive integers. If the domain is the vertex set then the labeling is call vertex labeling, if the domain is the edge set then the labeling is call edge labeling. Define a labeling $\boldsymbol{\phi: V \cup E \to \{1, 2, \dots k\}}$ as a vertex irregular total $\boldsymbol{k}$ −labeling if for every two different vertices $\boldsymbol{x}$ and $\boldsymbol{y}$ the vertex-weight $\boldsymbol{wt_\phi(x) \neq wt_\phi(y)}$ where the vertex-weigth is defined by $\boldsymbol{wt_\phi(x) = \phi(x) + \sum_{xy\in E}\phi(xy)}$. The minimum value of label $\boldsymbol{k}$ for which $\boldsymbol{G}$ has a vertex irregular total $\boldsymbol{k}$ −labeling is called the total vertex irregularity strength of $\boldsymbol{G}$ and denoted by $\boldsymbol{tvs(G)}$. We consider Hayat Tree Graph, a graph as symbol of Capital of Nusantara (IKN) which is ratified by president in June 2023. In this paper, we determined the total vertex irregularity strength of Hayat Tree Graph.
Keywords: *Hayat tree, Irregularity Strength, Total Vertex Irregulairty Strength, Vertex Irregular Total Labeling, Tree.*

1. INTRODUCTION

Graph labeling is a branch of graph theory which is widely used in various scientific fields, its applications are currently widely used in the fields of Information Engineering, Communication Networks, chemistry and so on. In 2007, Hedge in his paper [5] stated that the labeling of a graph was used in determining the crystal structure processed from X-Ray Diffraction (XRD) data. In 2016, Krishna in his paper [2] stated that the security system in a building can be arranged based on graphic labeling so that all areas, especially vulnerable areas, can be covered properly. The paper states that the security system in a building can be arranged based on graphic labeling so that all areas, especially vulnerable areas, can be covered properly. Graph labeling can determine the inventory system between distributors and stock regarding the amount of inventory and demand for existing goods or services (Krishnaa, 2016). Prasana, Sravati, and Sudhakar [8] said that graph labeling also has a big contribution in communication networks, both wireless and wired. In their paper the authors also say that graph labeling is used to determine various channels in communication networks. Varkey [14] further said that graph labeling is also used to analyze problems in Mobile Ad Hoc networks, such as connectivity, scalability, routing, network modeling and simulation. From the various applications that utilize graph labeling, it can be said that graph labeling has a big role in the current digital era, so it has the potential to develop more rapidly both for developing mathematical science. In this paper, the author will discuss the very popular labeling of a graph, namely labeling total irregular points. This labeling was first introduced by Baca et al (2007). The following is a definition of total irregular labeling of points on a graph.

Baca et al [3] defined a labeling $\phi: V \cup E \to \{1,2,\dots,k\}$ to be a vertex irregular total k −labeling if for every two different vertices x and y then the vertex weight $wt_\phi(x) \neq wt_\phi(y)$ where the vertex-weight is $wt_\phi(x) = \phi(x) + \sum_{xy\in E}\phi(xy)$. The minimum value of k for which G has a vertex irregular total k −labeling is defined as the total vertex irregularity strength of G and denoted by $tvs(G)$.

Baca et al. proved that if a tree T with n pendant vertices and no vertices of degree 2, then $\left\lceil\frac{n+1}{2}\right\rceil \leq tvs(T) \leq n$. Furthermore, they gave a lower bound and an upperbound on the total vertexx irregular strength for any graph G with v vertices and e edges, minimum degree δ dan maximum degree Δ, $\left\lceil\frac{|V|+\delta}{\Delta+1}\right\rceil \leq tvs(G) \leq |V| + \Delta - 2\delta + 1$. In this paper, they also determined the total vertex irregularity strength of cycles, stars, and complete graphs, that is, $tvs(C_n) = \left\lceil\frac{n+2}{3}\right\rceil$, $tvs(K_{1,n}) = \left\lceil\frac{n+1}{2}\right\rceil$ and $tvs\ (K_n) = 2$. Nurdin et al. [6] proved the following lower bound of tvs for any graph G.

Theorem 1.1. Let G be a connected graph having n_i vertices of degree $i (i = \delta, \delta+1, \delta+2, \dots, \Delta)$ where δ and Δ are the minimum and the maximum degree of G, respectively. Then,

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$$tvs \geq \max\left\{\left\lceil\frac{\delta + n_\delta}{\delta + 1}\right\rceil, \left\lceil\frac{\delta + n_\delta + n_{\delta+1}}{\delta + 2}\right\rceil, \dots, \left\lceil\frac{\delta + \sum_{i=\delta}^{\Delta}(n_i)}{\Delta + 1}\right\rceil\right\}$$

Nurdin et. Also give this following conjecture.

Conjecture: 1.2 [6] Let G be a connected graph having n_i vertices of degree $i(i = \delta, \delta + 1, \delta + 2, \dots, \Delta)$ where δ and Δ are the minimum and the maximum degree of G, respectively. Then,

$$tvs(G) = max\left\{\left\lceil\frac{\delta + n_\delta}{\delta + 1}\right\rceil, \left\lceil\frac{\delta + n_\delta + n_{\delta+1}}{\delta + 2}\right\rceil, \dots, \left\lceil\frac{\delta + \sum_{i=\delta}^{\Delta}(n_i)}{\Delta + 1}\right\rceil\right\}$$

Conjecture 1.2 has been verified by several authors for several families of graphs [7, 9,11, 12] and [13]. Another paper that discuss about total vertex irregularity of graphs are [1,4,10]

2. RESEARCH METHOD

The methods used in this research are as follows:

1. Determination of the lower limit of the irregularity value. This method is carried out by studying The structure of a graph. This study is needed to determine the position of the points and edges that will be labeled in such a way that the resulting weights are different.
2. Determination of the upper limit of the irregularity value. This method is done by constructing a labeling of the graph under study is such that the labeling is an irregular total labeling of points. Construction was carried out with pay attention to the degrees of the vertex on the graph. Label the vertex and edges on the graph with total k-labeling irregularity of the vertex in such a way that the vertex with smaller degrees are obtained smaller weight.

3. MAIN RESULTS

Definition 3.1. Let $G = C_{17} \odot K_1 + P_h$ be a hayat tree graph where the set of vertices and edges as follows:

$V(C_{17} \odot K_1 + P_h) = \{y_i, 1 \leq i \leq 11\} \cup \{y'_i, 1 \leq i \leq 4\} \cup \{y''_i, 1 \leq i \leq 4\} \cup \{x\}$ and

$E(C_{17} \odot K_1 + P_h) = \{v_i z_i, 1 \leq i \leq 17\} \cup \{v_i v_j, 1 \leq i \leq 16, 2 \leq j \leq 17\} \cup \{v_1 x\} \cup \{v_{17} x\} \cup \{y_i y'_j, 2 \leq i \leq 5 \cup i = 9, 1 \leq j \leq 4\} \cup \{x y'_2\} \cup \{y_i y''_k, i = 1 \cup 6 \leq i \leq 8 \cup 10 \leq i \leq 11, 1 \leq k \leq 4\} \cup \{y'_j y''_k, 1 \leq j \leq 4, 1 \leq k \leq 2\} \cup \{y''_j y''_k, 1 \leq j \leq 2, 3 \leq k \leq 4\}$.

For illustration, look at the hayat tree graph $C_{17} \odot K_1 + P_h$ in Figure 1 below:

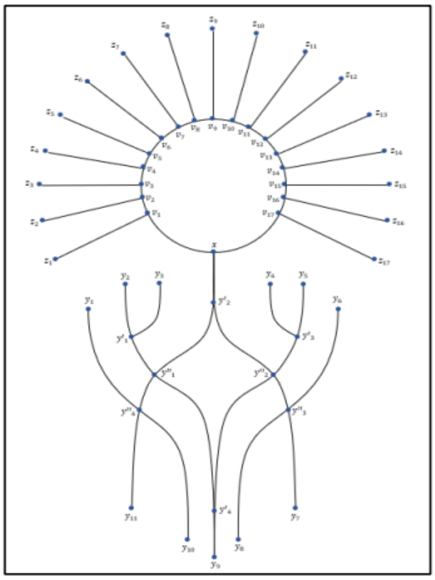

Figure 1. Hayat tree graph $C_{17} \odot K_1 + P_h$

Theorem 3. 1. *Let T be a hayat tree graph where* $C_{17} \odot K_1 + P_h$ *with n vertices,* $C_{17} \odot K_1 + P_h$ *then* $tvs(C_{17} + P_h) = \left\lceil \frac{n+1}{2} \right\rceil$.

PROOF.

1. Hayat tree graph is union of sun graph and tree graph. The vertex of sun graph degree one is 17 vertices and the vertices of tree graph is 11 vertices. The sum of vertices degree one is 28 vertices. Vertex labeling start with one and edge labeling start with 1. To determine the lower bound the weights by adding the vertex and edge labels. So the weight of the first point starts from 2. By Therefore, the weight of each point is different, it is proven that the total irregular point strength is $tvs(C_{17} \odot K_1 + P_h) \geq \left\lceil \frac{n+1}{2} \right\rceil$.

2. To prove the upper bound, we construct a labeling $C_{17} \odot K_1 + P_h$
   $V(C_{17} \odot K_1 + P_h) = \{v_1, v_2, \dots, v_{17}, z_1, z_2, \dots, z_{17}, y_1, y_2, \dots, y_{11}, y'_1, y'_2, \dots, y'_4, y''_1, y''_2, \dots, y''_4\}$
   $E(C_{17} \odot K_1 + P_h)$
   $$= \left\{ \begin{matrix} v_1z_1, v_2z_2, \dots, v_{17}z_{17}, v_1v_2, v_2v_3, \dots, v_{16}v_{17}, v_1x, v_{17}x, y_2y'_1, xy'_2, y_4y'_3, y_5y'_3, y_9y'_4, \\ y_1y''_4, y_6y''_3, y_7y''_3, y_8y''_3, y_{10}y''_4, y_{11}y''_4, y'_1y''_1, y'_2y''_1, y'_2y''_2, y'_3y''_2, y'_4y''_2, \\ y'_4y''_1, y''_1y''_4, y''_2y''_3 \end{matrix} \right\}$$

with x is connecting point between graph $C_{17} \odot K_1$ and P_h.

We construct hayat tree graph C_{17} with $\phi(x)$ is vertex labeling where $\phi(x_1) = 1, \phi(x_i) = \left\lceil \frac{n+1}{2} \right\rceil$ for all $x_i, i = 1,2,3, \dots ,28$. Furthermore $\phi(e_x)$ is edge labeling with $\phi(e_{x1}) = 1, \phi(e_{x_i}) = \left\lceil \frac{n+1}{2} \right\rceil$ for all $e_{x_i}, i = 1,2, \dots ,28$. Therefore, the weight of each point is different, it is proven the total irregular point strength is $tvs(C_{17} \odot K_1 + P_h) = \left\lceil \frac{n+1}{2} \right\rceil$ □

4. CONCLUSIONS

Let G be a simple graph with a set of vertices V(G) and a set of egdes E(G). Define a labeling $\phi: V \cup E \rightarrow \{1,2, \dots k\}$ as a vertex irregular total k −labeling or called tvs (G) is the smallest positive integer k of G. This paper formulates the vertex irregular total in the hayat tree graph $C_{17} \odot K_1 + P_h$ which produces tvs for degree one is $tvs(C_{17} + P_h) = \left\lceil \frac{n+1}{2} \right\rceil$ for all $x_i, i = 1,2, \dots ,28$.

Problem. Invsetigate the total edge irregularity strength of Hayat Tree Graph.

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