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THE CHROMATIC NUMBER FOR DELONIX REGIA AND PLUMERIA FLOWER GRAPHS

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Abstract
Let $\boldsymbol{G} = (\boldsymbol{V}, \boldsymbol{E})$ be a simple graph. A vertex k-coloring of a graph $\boldsymbol{G}$ is a labeling function $\boldsymbol{c}: \boldsymbol{V}(\boldsymbol{G}) \to \boldsymbol{T}$, where $|\boldsymbol{T}| = \boldsymbol{k}$ and it is proper if the adjacent vertices have different labels. A graph is $\boldsymbol{k}$ −colorable if it has a proper $\boldsymbol{k}$ −coloring. The chromatic number $\boldsymbol{\chi}(\boldsymbol{G})$ is the smallest such that $\boldsymbol{k}$ such that there exist a proper $\boldsymbol{k}$ −coloring of $\boldsymbol{G}$. This article investigated the chromatic number for Delonix Regia Flower ($\boldsymbol{DRF_n}$) and Plumeria Flower (PLF_n). The results showed that the chromatic number for Delonix regia flower ($\boldsymbol{DRF_n}$) is $\boldsymbol{\chi}(\boldsymbol{DRF_n}) = \boldsymbol{4}$ for $\boldsymbol{n} \geq \boldsymbol{4}$. Furthermore, the chromatic number for Plumeria Flower Graph $\boldsymbol{\chi}(\boldsymbol{PLF_n}) = \boldsymbol{4}$ for $\boldsymbol{n}$ is odd, and $\boldsymbol{\chi}(\mathbf{PLF}_{\boldsymbol{n}}) = \boldsymbol{3}$ for $\boldsymbol{n}$ is even.

Keywords: *Chromatic number, graph coloring, vertex coloring, graph operations, Delonix Regia graph, Plumeria flower graph*

1. INTRODUCTION

Mathematics is a branch of science that is developing very rapidly and most widely applied in everyday life. One of the branches the mathematical science that is developing rapidly is graph theory. Graph theory first introduced in 1736 by mathematician Leonhard Euler who applied it to the Konigsberg bridge problem, namely whether it is possible to pass through four regions connected by seven bridges so that each bridge is crossed exactly once. Bridge Problems Konisberg can be expressed in a graph by defining the four regions as vertex and the seven bridges as sides connecting the pair corresponding vertex.
In general, a graph is a pair of sets (V, E) where V is a non empty set of vertices and it's can be written as $V = \{v_1, v_2, v_3, \dots, v_n\}$ and E is a set of edges that connects a pair of vertices in the graph and it's written as $E = \{e_1, e_2, e_3, \dots, e_n\}$ [7]. One of the developing topics in graph theory is graph coloring that assigning colors to the vertices or edges of a graph. According to [1] there are 3 types of colorings in graphs that is vertex coloring, edge coloring, and region coloring.

The chromatic number indicates the minimum number of colors needed to color the vertices of a graph such that no two vertices are adjacent (directly related) have the same color. The chromatic number is denoted by $\chi(G)$.

According to [5], a k −coloring of a graph is a labeling $c: V(G) \to T$ where $|\mathrm{T}| = k$ and it is proper if the adjacent vertices have different labels. A graph is k −colorable if it has a proper k −coloring. The chromatic number $\chi(\mathrm{G})$ is the least k such that G is k −colorable. Irwanto et al. [10] has determined the chromatic number of special graphs. Saifudin et al. [17] has determined vertex coloring in special graphs. A number of other researchers have also determined the chromatic number for several types graphs [12], [8], [14], [15], [16], [11], [6], [2], [13], [4], [3], [9]. Therefore, it is done further research to discuss chromatic numbers in other graphs. In this research, we will study vertex coloring operations on Delonix regia flower graph (DRF_n) and Plumeria flower graphs (PLF_n) use the Greedy algorithm. There is upper bound for chromatic number for any graph G.

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Theorem 1. *If G is a simple graph, then the chromatic coloring number of its vertices $\chi(G)$ falls on this interval* $\chi(G) \leq \Delta(G) + 1$.

The Delonix regia flower graph (DRF_n) for $n \geq 4$ is a graph formed from identifying three graphs that is Anti-prism graph (AP_n), wheel graph (W_n), and helmet graph (H_n). Furthermore, define the Plumeria flower (PLF_n) for $n \geq 4$ is a graph formed from identifying three graphs that is prism graph (H_n), wheel graph (W_n), and fan graph (F_n).

2. RESEARCH METHOD

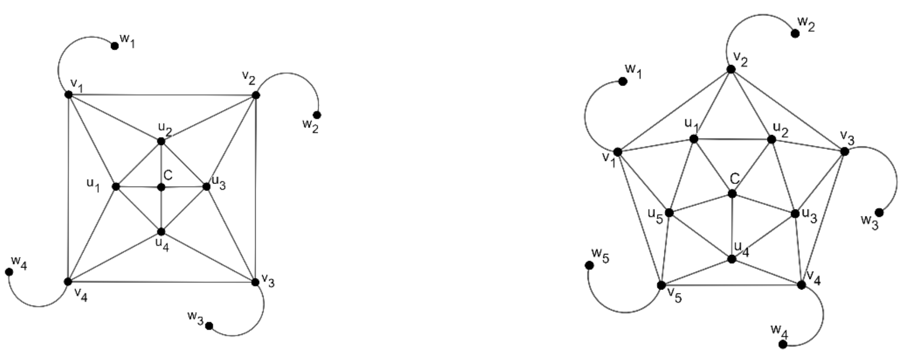
One of the algorithm that can be used to coloring the vertices is Greedy Algorithm. The steps are as follows :

1. Select a starting vertex
   Start by selecting a vertex as the starting vertex. The selection of the starting vertex should follow the predefined notation for vertex in the graph.
2. Color the starting vertex
   Assign color 1 to the starting vertex, then proceed to the other vertices. Make sure that there are no adjacent vertices have the same color.
3. Color the other vertices
   Continue by giving color 2 to vertices that are not yet colored, but make sure that no two adjacent vertices have the same color.
4. Continue with the same technique
   Apply the same technique by using a larger color one level above the previous color, and make sure that no two adjacent vertices have the same color.
5. Prove the upper and lower bound of chromatic number
   Ensure that the number of colors used does not exceed $\Delta(G) + 1$, where $\Delta(G)$ is the maximum degree of the graph G (the maximum number of neighbors of a vertex in the graph).

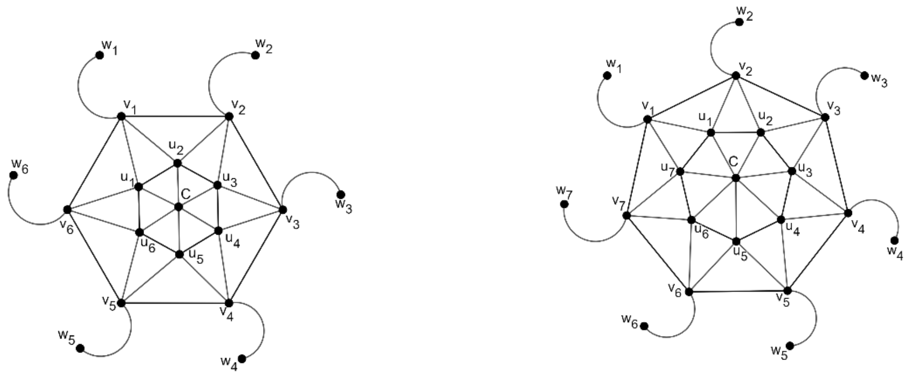
3. RESULT AND DISCUSSION

The results presented in this section provide the chromatic number of Delonix regia and Plumeria flower of graphs. Firstly, define the Delonix regia flower graph as follows.

Definition 1. Delonix regia flower graph denoted by (DRF_n) is a graph from joint graph operations of Anti-prism graph (AP_n), wheel graph *(W_n)*, and helmet graph (H_n). Delonix regia flower graph has the number of vertices $|V(DRF_n)| = (3n + 1)$ for $n \geq 4$, with n vertices of degree 1, $2n$ vertices of degree 5, and 1 vertex of degree n and has the number of edges $|E(DRF_n)| = 6n$. Figure 1 ilustrated Delonix regia flower graph.



(a) (b)



(c) (d)

Figure 1. Delonix regia flower graph (DRF_n) with $n \geq 4$. (a) DRF_4, (b) DRF_5, (c) DRF_6, (d) DRF_7.

This following theorem give the exact value of the chromatic number of Delonix Regia flower graph (DRF_n) for $n \geq 4$. To prove the lower bound, we use Theorem 1, and to obtain the upper bound, we construct a labeling by defining a function f as follows:

Theorem 2. Let $G = (DRF_n)$ be the Delonix Regia Flower Graph. The *chromatic number of the graph Delonix regia flower* (DRF_n) *with* $n \geq 4$ *is* $\chi(DRF_n) = 4$.

Proof.
There are vertices and edges set of Delonix regia flower.

$V(\mathrm{DRF}_n) = \{C, u_i, v_i, w_i | 1 \leq i \leq n\}$ and set of edges as follows.

$E(\mathrm{DRF}_n) = \{Cu_i | 1 \leq i \leq n\} \cup \{u_i u_{i+1} | 1 \leq i \leq n-1\} \cup \{u_n u_1\} \cup \{u_i v_i | 1 \leq i \leq n\} \cup \{u_{i+1} v_i | 1 \leq i \leq n-1\} \cup \{u_1 v_n\} \cup \{v_i v_i + 1 | 1 \leq i \leq n-1\} \cup \{v_n v_1\} \cup \{v_i w_i | 1 \leq i \leq n\}$.

where $p = |V| = 3n + 1$ and $q = |E| = 6n$. We will show that $\chi(DRF_n) = 4$ by considering two cases below.

Case 1. We show that $\chi(DRF_n) \geq 4$ by using Theorem 1,

$$\chi(DRF_n) \leq \Delta(DRF_n) + 1$$

$$\chi(DRF_n) \leq n + 1$$

$$4 \leq n + 1.$$

Thus, it satisfies $\chi(DRF_n) \leq n + 1, n \geq 4$ where n is odd.

Case 2. We show that $\chi(DRF_n) \leq 4$ by define a coloring function f as follow

i. $f(C) = 1$

ii. $f(u_i) = \begin{cases} 2;\ 1 \leq i \leq n \text{ and i odd} \\ 3;\ 1 \leq i \leq n \text{ and i even} \\ 4;\ \quad i = n \text{ and i odd} \end{cases}$

iii. $f(v_i) = \begin{cases} 1;\ 1 \leq i \leq n \text{ and i odd} \\ 4;\ 1 \leq i \leq n \text{ and i even} \\ 2;\ \quad i = n \text{ and i odd} \end{cases}$

iv. $f(w_i) = 3;\ 1 \leq i \leq n.$

It is proved that the Delonix regia flower graph has chromatic number $\chi(DRF_n) = 4$ and satisfies Theorem 1 that is $\chi(DRF_n) \leq \Delta(DRF_n) + 1$. This following Figure 2 ilustrated the coloring vertice for Delonix regia flower graph where $\chi(\mathrm{DRF}_n) = 4$.

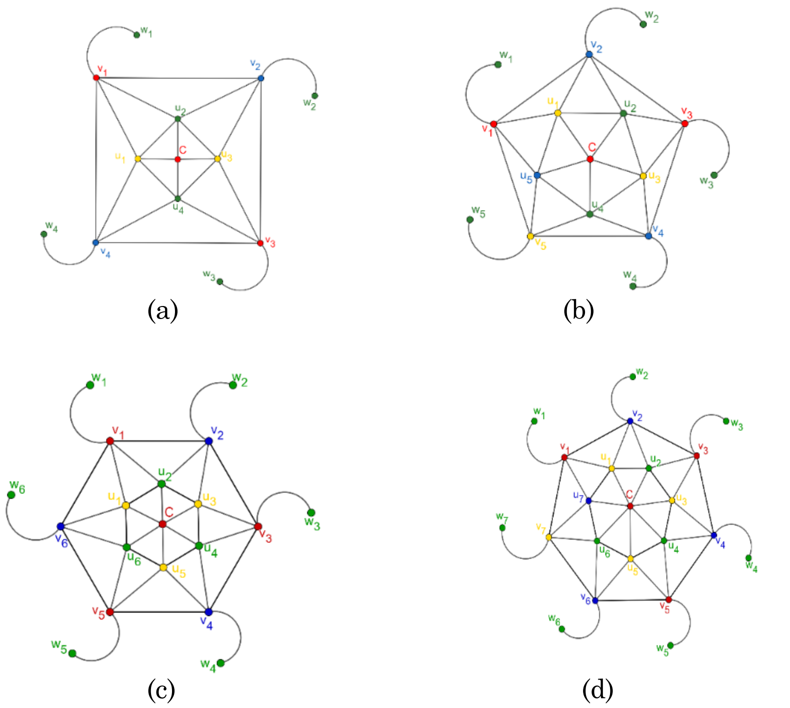
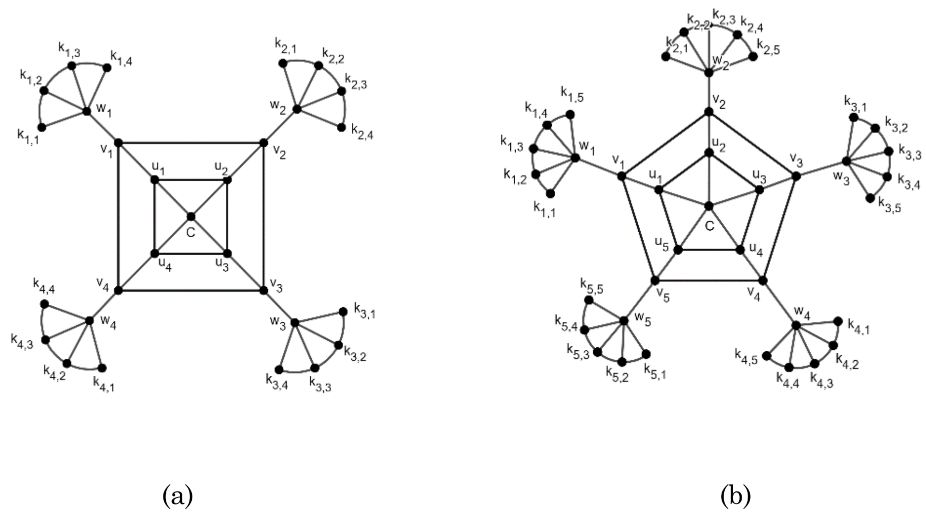
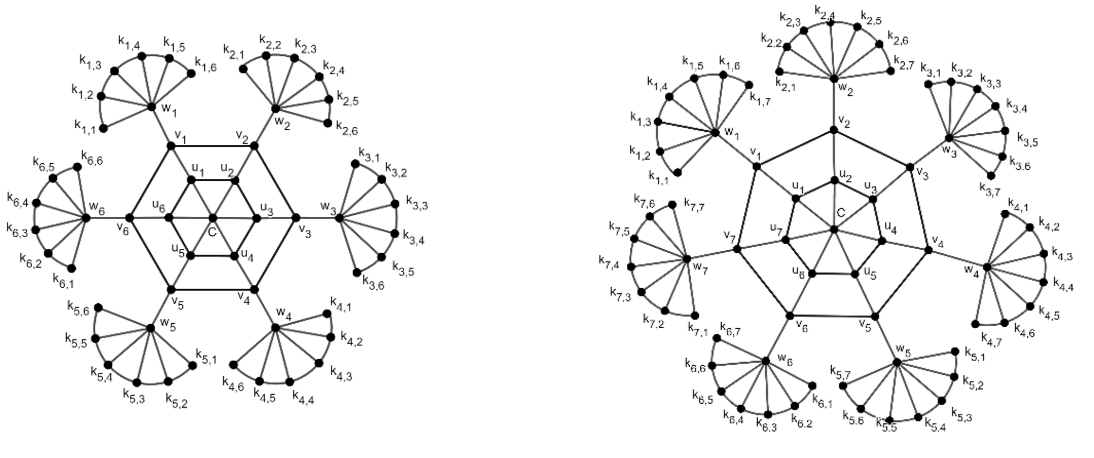


Figure 2. Delonix regia flower graph (DRF_n) with $n \geq 4$.
(a) DRF_4, (b) DRF_5, (c) DRF_6, (d) DRF_7.

Definition 2. The graph Plumeria Flower which is denoted by (PLF_n) is a graph formed by joint three classes of graph that is the prism graph (H_n), the wheel graph (W_n), and fan graph (F_n). The graph Plumeria Flower has the number of vertices $|V(\mathrm{PLF}_n)| = n(n+3)+1$ vertices for $n \geq 4$, with $2n$ vertices of degree 2, $n(n-2)$ vertices of degree 3, n vertices of degree $n+1$, $2n$ vertices of degree 4 and 1 vertex of degree n and has the number of edges $|E(\mathrm{PLF}_n)| = 2n^2 + 4n$.

This following Figure 3 ilustrated the plumeria flower graph (PLF_n).





(c) (d)

Figure 3. The graph Plumeria flower (P_n) with $n \geq 4$. (a) PLF_4, (b) PLF_5, (c) PLF_6, (d) PLF_7.

This following theorem give the exact value of the chromatic number of Plumeria flower graph (PLF_n) for n is odd and $n \geq 4$. To prove the lower bound, we use Theorem 1, and to obtain the upper bound, we construct a labeling by defining a function g as follows:

Theorem 3. Let $G = (\text{PLF}_n)$ be the Plumeria Flower Graph. The *Chromatic number of Plumeria flower graph* (PLF_n) *with* $n \geq 4$ *and n is odd, then* $\chi(\text{PLF}_n) = 4$.

Proof.

The Plumeria flower is a graph that has a set of vertices $V(PLF_n)$ and edges $E(PLF_n)$ as follows.

$V(\text{PLF}_n) = \{C, u_i, v_i, w_i | 1 \leq i \leq n\} \cup \{k_{1,j}, k_{2,j}, k_{3,j}, \dots, k_{n,j} | 1 \leq j \leq n\}$

$E(\text{PLF}_n) = \{C, u_i | 1 \leq i \leq n\} \cup \{u_i u_{i+1} | 1 \leq i \leq n-1\} \cup \{u_n u_1\} \cup \{u_i v_i | 1 \leq i \leq n\} \cup \{v_i v_{i+1} | 1 \leq i \leq n-1\} \cup \{v_n v_1\} \cup \{v_i w_i | 1 \leq i \leq n\} \cup \{w_1 k_{1,j} | 1 \leq j \leq n\}, \{w_2 k_{2,j} | 1 \leq j \leq n\}, \dots, \{w_n k_{n,j} | 1 \leq j \leq n\} \cup \{k_{1,j} k_{1,j+1} | 1 \leq j \leq n-1\}, \{k_{2,j} k_{2,j+1} | 1 \leq j \leq n-1\}, \dots, \{k_{n,j} k_{n,j+1} | 1 \leq j \leq n-1\}$,

where $p = |V| = n(n+3) + 1$ and $q = |E| = 2n^2 + 4n$.

The chromatic number of the Plumeria flower graph with odd n is 4 with the maximum degree being $n+1$. $\chi(\text{PLF}_n) = 4$ if and only if $\chi(\text{PLF}_n) \geq 4$ and $\chi(\text{PLF}_n) \leq 4$. First, we show that $\chi(\text{PLF}_n) \geq 4$ using Theorem 1, the chromatic number of the Plumeria flower graph for n odd as follow.

$$\chi(\text{PLF}_n) \leq \Delta(\text{PLF}_n) + 1$$

$$\chi(\text{PLF}_n) \leq n + 1 + 1$$

$$\chi(\text{PLF}_n) \leq n + 2$$

$$4 \leq n + 2.$$

Thus, it satisfies $\chi(\text{PLF}_n) \leq n + 2, n \geq 4$ where n is odd. Furthermore, to coloring the vertices, we define the function g as follow :

i. Color its vertices with:

$$g(C) = 1;\ 1 \leq i \leq n,$$

$$g(u_i) = \begin{cases} 2\ ;\ 1 \leq i \leq n \text{ and } i \text{ odd} \\ 3\ ;\ 1 \leq i \leq n \text{ and } i \text{ even} \\ 4\ ;\qquad i = n \text{ and } i \text{ odd} \end{cases}$$

$$g(v_i) = \begin{cases} 2\ ;\ 1 \leq i \leq n \text{ and } i \text{ even} \\ 3\ ;\ 1 \leq i \leq n \text{ and } i \text{ odd} \\ 1\ ;\qquad i = n \text{ and } i \text{ odd} \end{cases}$$

$$g(w_i) = 4;\ 1 \leq i \leq n,$$

$$g(k_{i,j}) = \begin{cases} 2;\ 1 \leq j \leq n \text{ and } j \text{ odd} \\ 3;\ 1 \leq j \leq n \text{ and } j \text{ even} \end{cases}$$

It is proven that the Plumeria flower graph for odd n has chromatic number $\chi(\text{PLF}_n) = 4$ and satisfies $\chi(\text{PLF}_n) \leq \Delta(\text{PLF}_n) + 1$. Based on Theorem 3, the vertex coloring for Plumeria flower graph for odd n is illustrated in Figure 4.

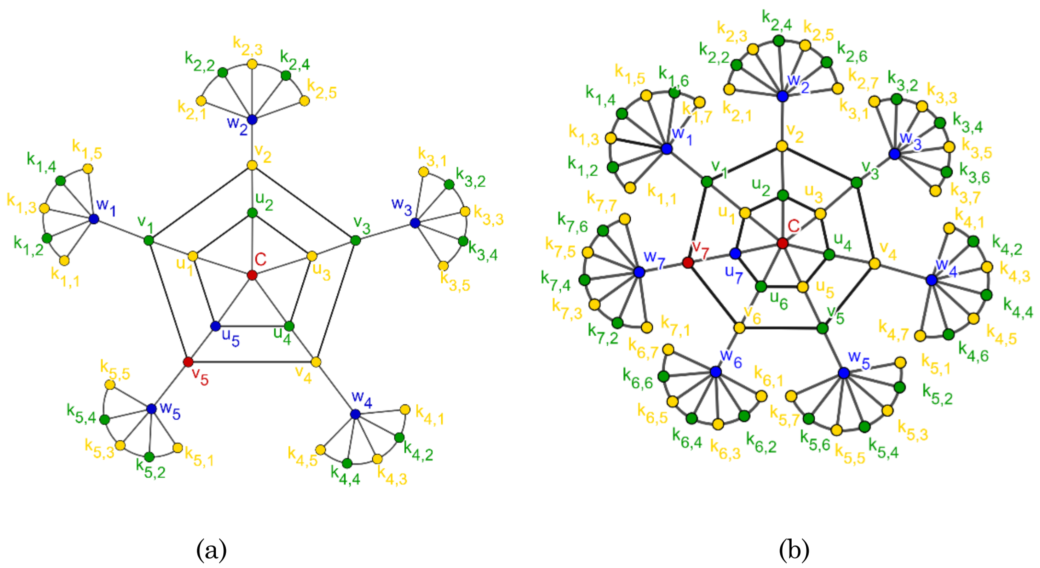


Figure 4. Vertex coloring of the Plumeria flower graph (PLF_n) with $n \geq 4$ for odd n. (a). PLF_5, (b) PLF_7

This following theorem give the exact value of the chromatic number of Plumeria flower graph (PLF_n) for n even and $n \geq 4$.

Theorem 4. Let $G = (\text{PLF}_n)$ be the Plumeria Flower Graph. The *Chromatic number of Plumeria flower graph* (PLF_n) *with* $n \geq 4$ *and* n *is even, then* $\chi(\text{PLF}_n) = 3$.

Proof.
The graph Plumeria flower is a graph that has a set of vertices

$V(\mathrm{PLF}_n) = \{C, u_i, v_i, w_i | 1 \le i \le n\} \cup \{k_{1,j}, k_{2,j}, k_{3,j}, \dots, k_{n,j} | 1 \le j \le n\}$

$E(\mathrm{PLF}_n) = \{C, u_i | 1 \le i \le n\} \cup \{u_i u_{i+1} | 1 \le i \le n-1\} \cup \{u_n u_1\} \cup \{u_i v_i | 1 \le i \le n\} \cup \{v_i v_{i+1} | 1 \le i \le n-1\} \cup \{v_n v_1\} \cup \{v_i w_i | 1 \le i \le n\} \cup \{w_1 k_{1,j} | 1 \le j \le n\}, \{w_2 k_{2,j} | 1 \le j \le n\}, \dots, \{w_n k_{n,j} | 1 \le j \le n\} \cup \{k_{1,j} k_{1,j+1} | 1 \le j \le n-1\}, \{k_{2,j} k_{2,j+1} | 1 \le j \le n-1\}, \dots, \{k_{n,j} k_{n,j+1} | 1 \le j \le n-1\}.$

where $p = |V| = n(n+3) + 1$ and $q = |E| = 2n^2 + 4n$.

The chromatic number of the Plumeria flower graph with n even is $\chi(\mathrm{PLF}_n) = 3$ if and only if $\chi(\mathrm{PLF}_n) \ge 3$ and $\chi(\mathrm{PLF}_n) \le 3$.

i. Using Theorem 1, the chromatic number of the Plumeria flower graph for even n is within the interval:

$$\chi(\mathrm{PLF}_n) \le \Delta(\mathrm{PLF}_n) + 1$$

$$\chi(\mathrm{PLF}_n) \le n + 1 + 1$$

$$\chi(\mathrm{PLF}_n) \le n + 2$$

$$3 \le n + 2.$$

Thus, it satisfies $\chi(\mathrm{PLF}_n) \le n + 2$, where $n \ge 4$ and n is even.

ii. Color its vertices with:

$$f(C) = 1;\ 1 \le i \le n,$$

$$f(u_i) = \begin{cases} 2;\ 1 \le i \le n, i\ odd \\ 3;\ 1 \le i \le n, i\ even \end{cases}$$

$$f(v_i) = \begin{cases} 2;\ 1 \le i \le n, i\ even \\ 3;\ 1 \le i \le n, i\ odd \end{cases}$$

$$f(w_i) = 1;\ 1 \le i \le n,$$

$$f(k_{i,j})_{(i=1,2,\dots,n)} = \begin{cases} 2;\ 1 \le j \le n, j\ odd \\ 3;\ 1 \le j \le n, j\ even \end{cases}$$

It is proven that the Plumeria flower graph for even n has chromatic number $\chi(\mathrm{PLF}_n) = 3$ and satisfies $\chi(\mathrm{PLF}_n) \le \Delta(\mathrm{PLF}_n) + 1$.

Based on Theorem 4, the vertex coloring of the Plumeria flower graph for even n is illustrated in Figure 5.

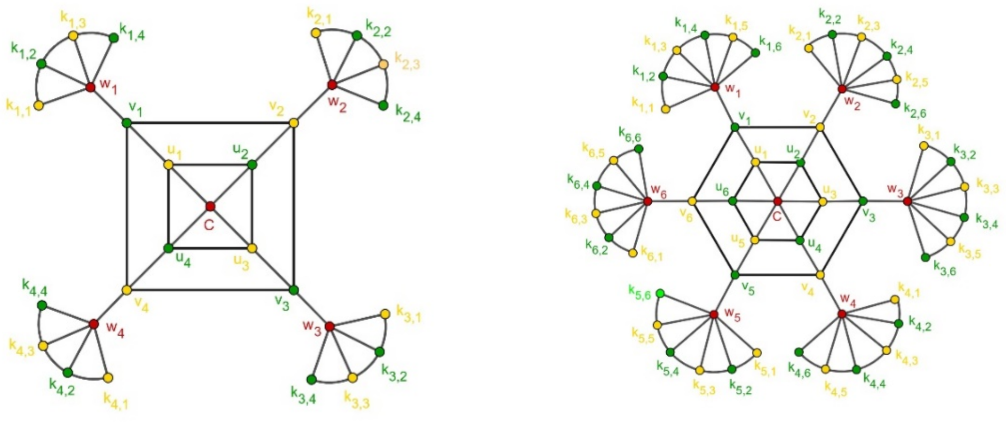


Figure 5. Vertex coloring of the Plumeria flower graph (PLF_n) with $n \geq 4$ for even n. (a) PLF_4, (b) PLF_6.

5. CONCLUSION

Based on the result of the analysis and discussion, it can be concluded that for the Delonix regia flower graph (DRF_n) where $n \geq 4$, the chromatic number obtained is $\chi(\mathrm{DRF}_n) = 4$, and for the Plumeria flower graph (PLF_n) where $n \geq 4$ and n is odd, then the chromatic number is $\chi(\mathrm{PLF}_n) = 4$. Furthermore, for n even, then the chromatic number is $\chi(\mathrm{PLF}_n) = 3$.

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