

Determining the Chromatic Number of a Modified Adenovirus Graph Using Greedy Algorithm

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Abstract

One of the vertex colorings that is a well-known research topic is chromatic number which requires that any two adjacent vertices have different colors. Adenovirus is a DNA virus that causes infections in the upper or lower respiratory tract, pharynx, gastrointestinal tract, and conjunctiva. Let $G = (AV_n)$ be the modified adenovirus graph. This graph is constructed from molecular biology data, where $V(G)$ represents a set of vertices that represented the elements such as virus DNA genes, DNA segments, and their variants, while $E(G)$ is the set of edges that describe overlapping interactions between segments or conflicts among them. This article discusses vertex coloring on the modified adenovirus graph (AV_n) using greedy. The chromatic number is the minimum number of colors used to solve the vertex coloring problem on the graph G and is denoted by $\chi(G)$. This study aims to construct the graph (AV_n) and determine the chromatic number of the graph (AV_n) using the greedy algorithm. The results show that greedy algorithm give the chromatic number for the modified adenovirus graph (AV_n) with $n \geq 4$ for even n is $\chi(AV_n) = 3$.

Keywords: Vertex coloring, chromatic number, modified adenovirus graph, greedy algorithm

1. INTRODUCTION

Graph theory studies the structural relationship between vertices and edges, with applications in various fields including biology, transportation networks, computer science, operations research, social sciences, mapping problems, and communication networks [5, p.19]. According to [5] and [23], the field of graph theory originated in 1736 when Swiss mathematician Leonhard Euler solved the Königsberg Bridge Problem by modeling it as a graph, where four land masses were represented as vertices and seven bridges as edges.

Graph theory has evolved, with one of its key developments being graph coloring, which is categorized into three types: vertex coloring, edge coloring, and region coloring [3], [6], [8], [15], [23]. The problem of graph coloring is related to the minimum number of colors used to solve the coloring problem on the graph, such that every two vertices and two adjacent regions and two incident edges have different colors as the basis for solving the graph coloring problem. According to [23, p.191], a graph is said to be colored with k colors if it has a proper k -coloring. The chromatic number $\chi(G)$ is the smallest value of k such that G can be colored with k colors, i.e., $\chi(G) = k$. In solving the problem of vertex coloring on a graph, when coloring all the vertices on the graph, it is impossible to use many colors to solve it, for that there are algorithms that have been introduced by various researchers, including the greedy algorithm. In this study, in addition to following the steps in each algorithm, the author also implemented it in a Pascal program to produce a solution.

Adenovirus is a DNA virus that causes infections in the upper or lower respiratory tract, pharynx, gastrointestinal tract, and conjunctiva. Adenovirus infections usually occur in young children because they have weak immune systems [13]. In [7] and [12] it is explained that adenovirus is medium-sized, namely 90-100 nm, consisting of hexons, icosahedral capsids, penton bases, fibers with knobs, and DNA genomes. It is known that viruses can grow rapidly, including adenoviruses. Adenoviruses are represented in a graph called the adenovirus modification graph. This graph is usually formed from molecular biology data with $V(G)$ being a set of vertices representing something like genes from viral DNA, DNA segments, and their

variants, while $E(G)$ is a set of edges that describe overlapping interactions between segments or conflicts between segments. The adenovirus modification graph is obtained by constructing a new graph from a prism graph (H_n) which is added with a center vertex c that is connected to all vertices on the inner cycle. In the outer cycle, pendant vertices are added that are directly connected to the center vertex. Next, amalgamate the vertices on the sandat graph (St_n) to the center vertex c , so that a new graph is formed, namely the modified adenovirus graph, which is denoted by (AV_n) .

In [14] and [16], the heuristic algorithm for vertex coloring problems has been discussed, including the greedy algorithm and the minimum number of colors used. Then, [1] has discussed the implementation of the greedy algorithm in graph coloring, namely vertex coloring in the sudoku game represented in the form of a graph. In [21], the implementation of the greedy algorithm has been studied in the coloring of a region consisting of 22 sub-districts represented as vertices and 41 edges connecting the sub-districts. In [10], the chromatic number of vertex coloring in special graphs and its operations, one of which is the prism graph, has been studied. Furthermore, in [19], the vertex coloring in the family of centripetal graphs including the sandat graph has been discussed. Then, in [11], the analysis of the differences between algorithms for vertex coloring problems in graphs has been studied. The discussion in [11] focuses on the analysis of three different algorithms for vertex coloring problems, namely the difference between the exact algorithm based on the backtracking algorithm and the heuristic algorithm, namely the greedy algorithm and the Welch-Powell algorithm. Until now, no one has studied the modified adenovirus graph. The purpose of this study is to determine the chromatic number of the modified adenovirus graph using the greedy algorithm. Therefore, researchers will develop a greedy algorithm in solving the vertex coloring problem in constructing a new graph, namely the modified adenovirus graph (AV_n) with $n \geq 4$ for even n .

2. PRELIMINARIES

The following definitions and theorems are used in this research

2.1 Graph Terminology

According to [2], [3], [5], [15] and [20], graphs G is a pair of sets $(V(G), E(G))$ consisting of a non-empty and finite set $V(G)$ called a vertex and $E(G)$ is a set of unordered pairs of two vertices in $V(G)$ and a possibly empty set called an edge. The number of vertices in $V(G)$ is called the order of G and is denoted by $p(G) = |V|$ while the number of edges in $E(G)$ is called the size of G and is denoted by $q(G) = |E|$. In a graph, there are relationships between objects, specifically vertex to vertex and vertex to edge, which are commonly referred to as adjacent and incidence, respectively. Let $e = (u, v)$ be the edge connecting vertices u and v . If $e = uv \in E(G)$, then vertices u and v said to be adjacent, vertex u and e and vertex v and e are said to be incident.

According to [3], [6], [23] each vertex has a degree, which refers to the number of edges adjacent to a vertex v_i . The number of edges incident to a vertex v is called the degree of v in the graph G and is denoted by $\deg v$ or $d(v)$. We denoted $\Delta(G)$ as maximum degree of the graph G . A vertex with degree one is called a pendant vertex. According to [9], [17], [22], amalgamation is an operation on a graph that generates a new graph by selecting one vertex from two or more graphs and merging them into one vertex. Let $\{G_i | i \in \{1, 2, \dots, n\}\}$ be a finite set of graphs, where each G_i has a fixed vertex v_{0i} called the terminal vertex. The amalgamation of vertices is formed by merging all the graphs G_i at vertex v_{0i} . According to [3], an adjacency matrix is a matrix that is used to represent a connections graph, where rows and columns correspond to vertices. The values in the matrix represent whether or not there is an edge connecting the two vertices. Suppose a graph G has n vertices, where $V(G) = \{v_1, v_2, \dots, v_n\}$. The adjacency matrix $A = [a_{ij}]$ of size $n \times n$ is defined as follows:

$$a_{ij} = \begin{cases} 1; & \text{if } v_i v_j \in (G) \\ 0; & \text{if } v_i v_j \notin (G) \end{cases}$$

2.2 Special Types of Graphs

In [10] and [19], the following are definitions of two special types of graphs: prism graph and sandat graph.

Definition 2.1. The prism graph, denoted by H_n , is the Cartesian product of a cycle graph and a path graph as $(C_n \times P_2)$, where C_n is a cycle graph with n vertices and P_2 is a path graph with 2 vertices. A prism graph (H_n) has a vertex set and edge set as follows :

$$V(H_n) = \{u_i, v_i \mid 1 \leq i \leq n\}$$

$$E(H_n) = \{u_i u_{i+1} \mid 1 \leq i \leq n-1\} \cup \{u_n u_1\} \cup \{u_i v_i \mid 1 \leq i \leq n\} \cup \{v_i v_{i+1} \mid 1 \leq i \leq n-1\} \cup \{v_n v_1\} ,$$

where $p = |V| = 2n$ and $q = |E| = 3n$.

Definition 2.2. The sandat graph, denoted by St_n has a vertex set $V(St_n) = \{c\} \cup \{y_i, y_{i,1}, y_{i,2} \mid 1 \leq i \leq n\}$ and an edge set $E(St_n) = \{cy_i, cy_{i,1}, cy_{i,2}, y_i y_{i,1}, y_i y_{i,2} \mid 1 \leq i \leq n\}$, where $p = |V| = 3n + 1$ and $q = |E| = 5n$. The sandat graph with $i = 3, 4, \dots, n$ is represented as follows:

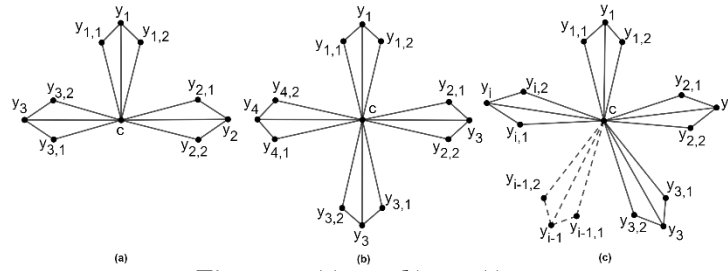


Figure 1. (a) St_3 , (b) St_4 , (c) St_n

2.3 Graph Coloring

According to [3], [6], [8], [23] in graph theory, graph coloring is a coloring technique for assigning colors to elements of a graph, such as vertices, edges, or regions. Each adjacent vertex, adjacent region, and incident edge must have different colors, using the minimum number of colors possible. Graph coloring is categorized into three types: vertex coloring, edge coloring, and region coloring. Vertex coloring on a graph G is a technique for assigning a color to each vertex on a graph G such that no two adjacent vertices have the same color.

The vertex coloring problem is closely related to the chromatic number, which represents the minimum number of colors required for a proper coloring. The vertex coloring on a graph is represented in Figure 2.

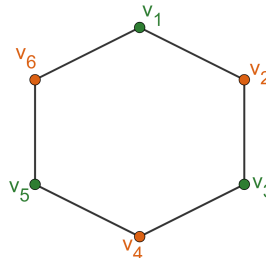


Figure 2. Vertex coloring with 2 colors

Based on Figure 2, since the graph G can be colored with 2 colors, the chromatic number of the graph G is $\chi(G) = 2$.

2.4. Chromatic Number

Coloring each vertex of a graph G with adjacent vertices having different colors is called the proper coloring of the graph G . The following is the definition of chromatic number according to [3], [5], [8], [23].

Definition 2.3. The minimum number of colors used to solve the problem of coloring vertices in a graph G such that every two adjacent vertices have different colors is called the chromatic number of the graph G , denoted by $\chi(G)$. A graph is said to be colorable with k colors if it has a proper k -colorings. The chromatic number $\chi(G)$ is the smallest k such that G can be properly colored with k colors. Therefore, $\chi(G) = k$.

Theorem 1. If $\Delta(G)$ is the maximum degree of a vertex in graph G , then:

$$\chi(G) \leq \Delta(G) + 1$$

PROOF. The proof of Theorem 1 is shown in [4, p.120] using a greedy algorithm.

2.5 Greedy Algorithm

In [2], [6], [14], [16], [21], and [23], the greedy algorithm has been discussed as a simple algorithm that makes locally optimal choices at each step without considering future consequences, hoping to achieve an overall optimal solution. At each step, multiple choices need to be evaluated to determine the best option. The fundamental principle of the greedy algorithm is “take what you can get now!”. The steps for finding a solution using the greedy algorithm are as follows:

- 1) Select a vertex in graph G to color and assign the first color to that vertex.
- 2) Select another vertex on the graph G that has not been colored, then color that vertex with the previous color.
- 3) Check the vertex; if it is adjacent and has the same color as the previous vertex, then assign a new color.
- 4) Repeat step 2 until all the vertices in the graph G are colored.

Given a graph G represented in Figure 3.

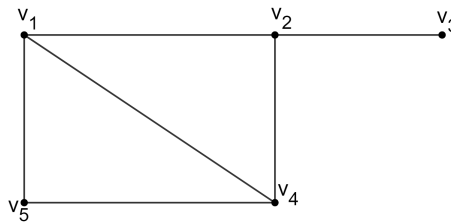


Figure 3. Graph G

Based on Figure 6, the colored graph G using the greedy algorithm is shown in Figure 4.

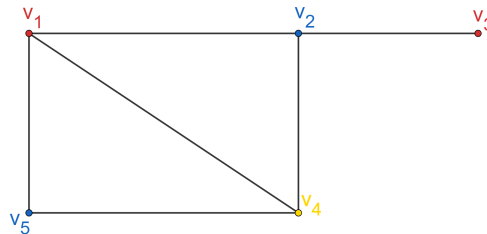


Figure 4. Vertex coloring solution using the greedy algorithm

Based on Figure 4, all vertices have been colored with different colors for adjacent vertices, so the chromatic number obtained is $\chi(G) = 3$.

The source code of Greedy Coloring is shown in Figure 5.

```

program GraphColoring;
uses crt, sysutils;
const
  nMAX = a; // Jumlah titik dalam graf hasil konstruksi
var
  AdjacencyMatrix: array[1..nMAX, 1..nMAX] of integer;
  Color: array[1..nMAX] of integer;
  n, i, j, maxColor: integer;
  startTime, endTime, elapsedTime: QWord;

procedure GreedyColoring;
var
  available: array[1..nMAX] of boolean;
  u, v, c: integer;
begin
  { Inisialisasi semua titik belum diberi warna }
  for u := 1 to n do
    Color[u] := 0;

  { Pewarnaan simpul pertama dengan warna pertama (warna 1) }
  Color[1] := 1;
  maxColor := 1;

  { Pewarnaan simpul lainnya }
  for u := 2 to n do
    begin
      { Tandai semua warna sebagai tersedia }
      for c := 1 to n do
        available[c] := true;

      { Tandai warna dari simpul tetangga sebagai tidak tersedia }
      for v := 1 to n do
        if (AdjacencyMatrix[u][v] = 1) and (Color[v] <> 0) then
          available[Color[v]] := false;
      { Pilih warna terkecil yang tersedia }
      for c := 1 to n do
        if available[c] then
          begin
            Color[u] := c;
            if c > maxColor then
              maxColor := c;
            break;
          end;
        end;
      end;
    end;

begin
  clrscr;
  n := nMAX;

  { Inisialisasi Matriks Adjacency }
  // Inisialisasi matriks adjacency untuk menggambarkan graf
  for i := 1 to VertexCount do
    for j := 1 to VertexCount do
      begin
        AdjacencyMatrix[i][j] := 0; // Ubah nilai berdasarkan graf yang diuji
      end;
    end;
  end;

```

Figure 5. The Pascal program source code for the greedy algorithm.

3. RESULT AND DISCUSSION

In this section we construct the modified Adenovirus Graph and determine the chromatic number with greedy algorithm

3.1. Modified Adenovirus Graph

Definition 3.1. The modified adenovirus graph is denoted by (AV_n) is obtained by constructing a new graph from a prism graph (H_n) which is added with a center vertex c that is connected to all vertices on the inner cycle. In the outer cycle, pendant vertices are added that are directly connected to the center vertex. Next, amalgamate the vertices on the sandat graph (St_n) to the center vertex c , so that a new graph is formed, namely the modified adenovirus graph. Modified adenovirus graph (AV_n) has an order $|V(AV_n)| = 2(3n + 1) - 1$ for $n \geq 4$, with $3n$ vertices of degree 2, n vertices of degree 3, $2n$ vertices of degree 4, and 1 vertex of degree $5n$, and has $|E(AV_n)| = 11n$.

The following is a comparison between the adenovirus and the modified adenovirus graph for $n = 4$, represented as follows:

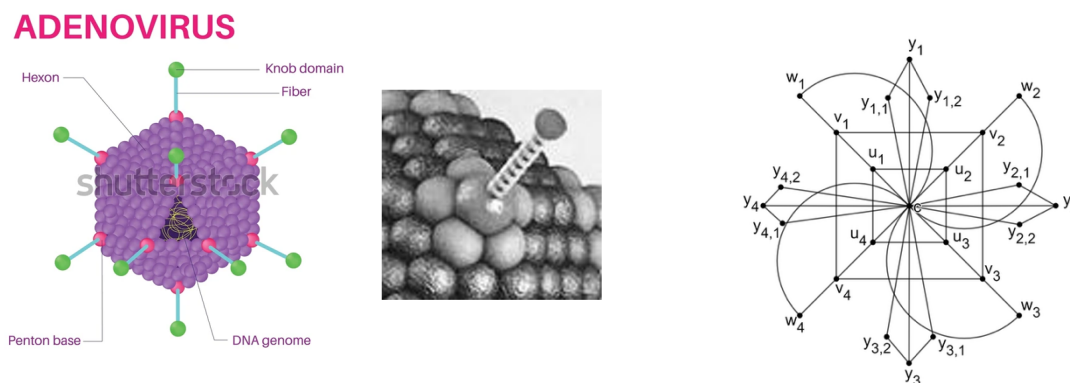
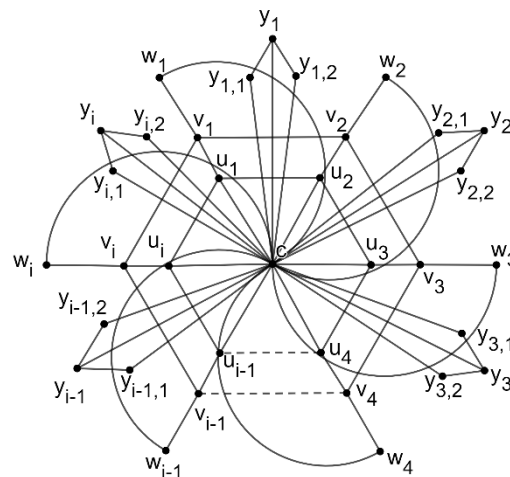


Figure 6. (a) Adenovirus dan (b) Modified adenovirus graph (AV_4)

The general modified adenovirus graph (AV_n) for $n \geq 4$ can be seen in Figure 7.



The modified adenovirus graph (AV_n) has vertex set $V(AV_n) = \{c, u_i, v_i, w_i \mid 1 \leq i \leq n\}$

$\cup \{y_i, y_{i,1}, y_{i,2} | 1 \leq i \leq n\}$ and edge set $E(AV_n) = \{c u_i | 1 \leq i \leq n\} \cup \{u_i u_{i+1} | 1 \leq i \leq n-1\} \cup \{u_n u_1\} \cup \{u_i v_i | 1 \leq i \leq n\} \cup \{v_i v_{i+1} | 1 \leq i \leq n-1\} \cup \{v_n v_1\} \cup \{v_i w_i | 1 \leq i \leq n\} \cup \{c w_i | 1 \leq i \leq n\} \cup \{c y_i, c y_{i,1}, c y_{i,2}, y_i y_{i,1}, y_i y_{i,2} | 1 \leq i \leq n\}$. Modified adenovirus graph (AV_n) with $p = |V| = 2(3n + 1) - 1$ and $q = |E| = 11n$.

Based on the definition and theory explained above, the next step is to determine the value of the chromatic number in the modified adenovirus graph (AV_n) with $n \geq 4$ for even n using the greedy algorithm. For this proof, the author only takes for $n = 4, 6, 8, 10, 12$. This is because the modified adenovirus graph (AV_n) with $n \geq 4$ for even n must have the same chromatic number.

The following modified graph of adenovirus (AV_n) is represented in Figure 9.

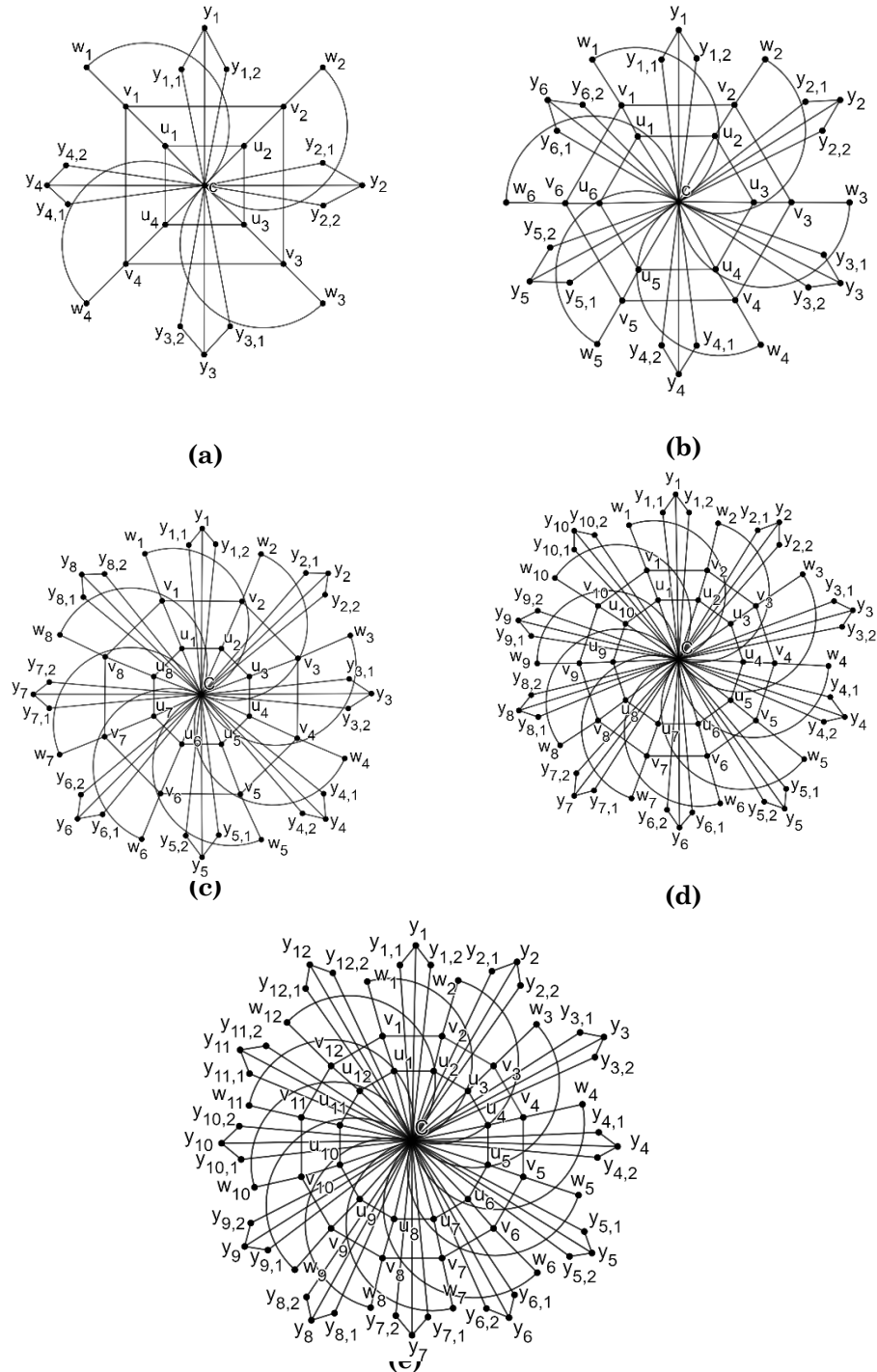


Figure 8. (a) AV_4 , (b) AV_6 , (c) AV_8 , (d) AV_{10} , (e) AV_{12}

Theorem 3.1. *Given $G = (AV_4)$ as the modified adenovirus graph which has $2(3n + 1) - 1$ vertices and $11n$ edges. The chromatic number of the modified adenovirus graph with $n \geq 4$ for even n is 3. Thus, $\chi(AV_n) = 3$.*

PROOF. It will be proven that $\chi(AV_n) = 3$ with $n \geq 4$ for even n using greedy algorithm.

The chromatic number on the modified adenovirus graph (AV_n) with $n \geq 4$ for even n is 3. $\chi(AV_n) = 3$ if and only if it satisfies:

- i. $\chi(AV_n) \geq 3$
- ii. $\chi(AV_n) \leq 3$

Proof.

- i. It will be shown that $\chi(AV_n) \geq 3$ by Theorem 2.12, the chromatic number of the modified adenovirus graph (AV_n) is in the interval:

$$\chi(AV_n) \leq \Delta(AV_n) + 1,$$

$$\chi(AV_n) \leq 5n + 1,$$

$$3 \leq 5n + 1.$$

Such that it satisfies $\chi(AV_4) \leq 5n + 1$, with $n \geq 4$ for even n .

- ii. It will be shown that $\chi(AV_n) \leq 3$ by defining the coloring function f as follows:

$$f(c) = 1 \quad ; 1 \leq i \leq n,$$

$$f(u_i) = \begin{cases} 2 & ; 1 \leq i < n \text{ and } i \text{ odd,} \\ 3 & ; 1 \leq i \leq n \text{ dan } i \text{ even,} \end{cases}$$

$$f(v_i) = \begin{cases} 1 & ; 1 \leq i < n \text{ dan } i \text{ odd,} \\ 2 & ; 1 \leq i \leq n \text{ dan } i \text{ even,} \end{cases}$$

$$f(w_i) = \begin{cases} 2 & ; 1 \leq i < n \text{ dan } i \text{ odd,} \\ 3 & ; 1 \leq i \leq n \text{ dan } i \text{ even,} \end{cases}$$

$$f(y_i) = 3 \quad ; 1 \leq i \leq n,$$

$$f(y_{i,j})_{(j=1,2)} = 2 \quad ; 1 \leq i \leq n.$$

Based on Theorem 2.12 and following the vertex coloring steps on the modified adenovirus graph (AV_n) with $n \geq 4$ for even n , it is proven that the modified adenovirus graph (AV_n) with $n \geq 4$ for even n , has a chromatic number of $\chi(AV_n) = 3$ and satisfies conditions i and ii.

The colored modified adenovirus graph (AV_n) with $n \geq 4$ for even n can be represented in Figure 10.

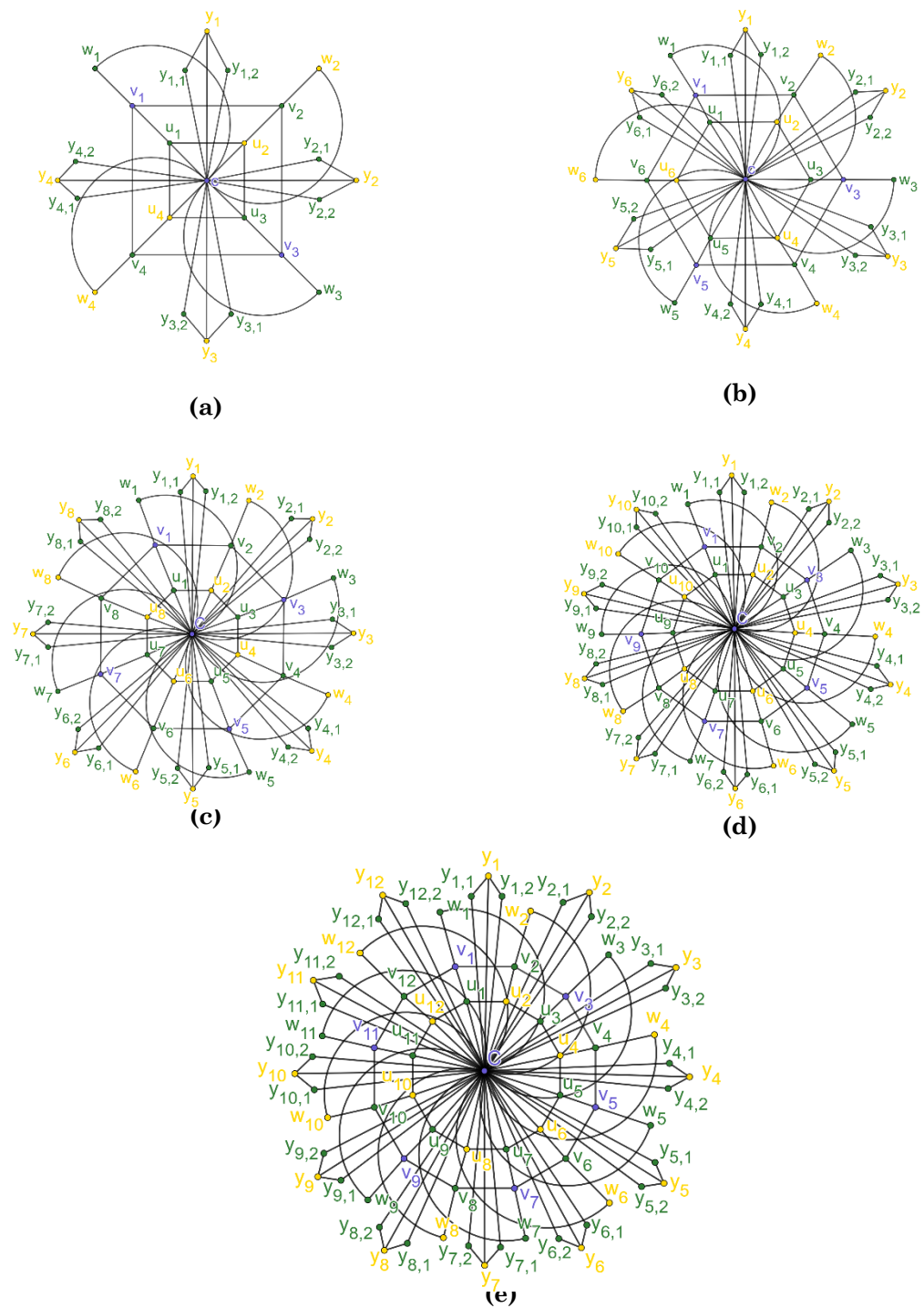


Figure 9. (a) AV_4 , (b) AV_6 , (c) AV_8 , (d) AV_{10} , (e) AV_{12}

The following are the results of the Pascal program in solving vertex coloring with the greedy algorithm represented in Figure 11.

```
Warna tiap titik:
Titik 1: Warna 1
Titik 2: Warna 2
Titik 3: Warna 3
Titik 4: Warna 2
Titik 5: Warna 3
Titik 6: Warna 1
Titik 7: Warna 2
Titik 8: Warna 1
Titik 9: Warna 2
Titik 10: Warna 2
Titik 11: Warna 3
Titik 12: Warna 2
Titik 13: Warna 3
Titik 14: Warna 2
Titik 15: Warna 3
Titik 16: Warna 2
Titik 17: Warna 2
Titik 18: Warna 3
Titik 19: Warna 2
Titik 20: Warna 2
Titik 21: Warna 3
Titik 22: Warna 2
Titik 23: Warna 2
Titik 24: Warna 3
Titik 25: Warna 2
Jumlah warna yang digunakan: 3
Waktu eksekusi: 0 milidetik
```

(a)

```
Titik 11: Warna 2
Titik 12: Warna 1
Titik 13: Warna 2
Titik 14: Warna 2
Titik 15: Warna 3
Titik 16: Warna 2
Titik 17: Warna 3
Titik 18: Warna 2
Titik 19: Warna 3
Titik 20: Warna 2
Titik 21: Warna 3
Titik 22: Warna 2
Titik 23: Warna 2
Titik 24: Warna 3
Titik 25: Warna 2
Titik 26: Warna 2
Titik 27: Warna 3
Titik 28: Warna 2
Titik 29: Warna 2
Titik 30: Warna 3
Titik 31: Warna 2
Titik 32: Warna 2
Titik 33: Warna 3
Titik 34: Warna 2
Titik 35: Warna 2
Titik 36: Warna 3
Titik 37: Warna 2
Jumlah warna yang digunakan: 3
Waktu eksekusi: 0 milidetik
```

(b)

```
Titik 23: Warna 3
Titik 24: Warna 2
Titik 25: Warna 3
Titik 26: Warna 2
Titik 27: Warna 3
Titik 28: Warna 2
Titik 29: Warna 2
Titik 30: Warna 3
Titik 31: Warna 2
Titik 32: Warna 2
Titik 33: Warna 3
Titik 34: Warna 2
Titik 35: Warna 2
Titik 36: Warna 3
Titik 37: Warna 2
Titik 38: Warna 2
Titik 39: Warna 3
Titik 40: Warna 2
Titik 41: Warna 2
Titik 42: Warna 3
Titik 43: Warna 2
Titik 44: Warna 2
Titik 45: Warna 3
Titik 46: Warna 2
Titik 47: Warna 2
Titik 48: Warna 3
Titik 49: Warna 2
Jumlah warna yang digunakan: 3
Waktu eksekusi: 0 milidetik
```

(c)

```
Titik 35: Warna 2
Titik 36: Warna 3
Titik 37: Warna 2
Titik 38: Warna 2
Titik 39: Warna 3
Titik 40: Warna 2
Titik 41: Warna 2
Titik 42: Warna 3
Titik 43: Warna 2
Titik 44: Warna 2
Titik 45: Warna 3
Titik 46: Warna 2
Titik 47: Warna 2
Titik 48: Warna 3
Titik 49: Warna 2
Titik 50: Warna 2
Titik 51: Warna 3
Titik 52: Warna 2
Titik 53: Warna 2
Titik 54: Warna 3
Titik 55: Warna 2
Titik 56: Warna 2
Titik 57: Warna 3
Titik 58: Warna 2
Titik 59: Warna 2
Titik 60: Warna 3
Titik 61: Warna 2
Jumlah warna yang digunakan: 3
Waktu eksekusi: 0 milidetik
```

(d)

```
Titik 47: Warna 2
Titik 48: Warna 3
Titik 49: Warna 2
Titik 50: Warna 2
Titik 51: Warna 3
Titik 52: Warna 2
Titik 53: Warna 2
Titik 54: Warna 3
Titik 55: Warna 2
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Titik 65: Warna 2
Titik 66: Warna 3
Titik 67: Warna 2
Titik 68: Warna 2
Titik 69: Warna 3
Titik 70: Warna 2
Titik 71: Warna 2
Titik 72: Warna 3
Titik 73: Warna 2
Jumlah warna yang digunakan: 3
Waktu eksekusi: 0 milidetik
```

(e)

Figure 10. (a) AV_4 , (b) AV_6 , (c) AV_8 , (d) AV_{10} , (e) AV_{12}

1. CONCLUSIONS

In this research, the implementation of the algorithm has been given. Based on the discussion above, the greedy algorithm is able to solve the problem of vertex coloring on the modified adenovirus graph (AV_n) with $n \geq 4$ for even n . The greedy algorithm is able to find the optimal solution with a very short execution time using 3 colors. Thus, the chromatic number of the modified adenovirus graph with $n \geq 4$ for even n , namely $\chi(AV_n) = 3$.

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