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Project Scheduling Analysis Using the CPM and PERT: A Case Study

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Abstract

This article discusses the scheduling analysis of a project using the critical path method (CPM) and the program evaluation and review technique (PERT) to find the project's critical path so that the overall project time can be determined. This problem aims to optimize the completion time of a project and minimize delays that cause project planning to be inconsistent with what is in the field based on activity time, activity implementation schedule, and project activity costs. This problem analysis is carried out with the help of Excel Solver software. The calculation results show that the optimal project completion time is obtained and the acceleration of project completion can be accelerated by selecting activities that can be accelerated with the project acceleration costs.

Keywords: *Project scheduling, critical path method, program evaluation and review technique*

1. PREFACE

Operations research is widely applied in solving management problems, one of which is often used in helping work on a project. Turner [9] describes a project as an undertaking in which human, financial, and material resources are organized in a new way to carry out a unique scope of work from given specifications, within cost and time constraints, thereby achieving profitable changes determined by quantitative and qualitative objectives. Chen and Hall [1] state that as much as 30% of world economic activity is organized as projects. Therefore, proper project management is needed in order to complete a project optimally.

In general, companies determine the time and cost of working on a project based on previous experience, and set time limits or deadlines. However, in certain cases, problems or obstacles may arise that cause planning to be inconsistent with project implementation in the field. To overcome this problem, management tools are needed in the form of methods related to planning and controlling costs and time on a project.

Critical path method and program evaluation and review technique are methods that are usually applied to help project planner to decide various optimization problems in project scheduling. Winston [10, p.433] explains that CPM can be used to determine the length of time needed to complete a project with a known duration of each activity. However, if the duration of an activity is not known with certainty, PERT can be used to estimate the probability that the project will be completed by the given deadline.

Research using the CPM and PERT methods to solve various problems of optimizing time and costs in project scheduling has been carried out a lot. Liu [8] uses the CPM and PERT methods to analyze construction schedule risks in a structural steel procurement project for buildings with as many as six work activities. Karabulut [5] conducts research by applying the Monte Carlo simulation method and the CPM/PERT method in planning a luxury villa construction project with a total of 17 activities and a total project duration of 186 days.

The author implements the CPM and PERT methods at PT. Transalindo Eka Persada, which is an ASME and MIGAS certified company, which has the scope of work in the design and manufacture of pressure vessels, as well as manufacture of process equipment packages and other steel construction. The author analyzes project scheduling in the manufacture of pressure vessels, namely Project 595-PTS-PJR-001 RB BRONANG GAS SEPARATOR (20MBD-003) with a contract value of IDR 618,000,000 and duration of completion this project is 160 days. This is done in order to be able to find out the optimal duration of project completion and to make all decisions in the scheduling process, and to be able to complete the project on time.

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In this article, the author uses the Excel Solver software to calculate the optimal project completion duration according to the CPM and PERT methods. This study uses data in the form of activity time, implementation schedule, and project costs. In part two, several research methods, which forms this study, will be explained. In part three, a discussion of the results of project scheduling analysis using the CPM and PERT methods will be explained. Then continued in part four by explaining the conclusions of the discussion of this article.

2. RESEARCH METHODS

2.1 Critical Path Method

The critical path method is a method used to determine the fastest time needed to complete a project through a deterministic approach or which reflects certainty. Heizer and Render [4] state that the CPM framework follows six basic steps with one of the main objectives being to determine the critical path. Zareei [11] states that the critical path length is equivalent to the total completion time of a project. There are several uses of the term time information related to critical paths in the work network, such as ES which is the early start or the earliest time the activity starts, EF which is the early finish or the earliest time the activity is finished, then LS which is the latest start or the latest time the activity starts, and LF is the latest finish or the latest time the activity is finished.

After knowing the time information for each activity, then it is necessary to determine the duration needed to complete a project with a two-pass process to find out the starting time and ending time for each activity.

Forward Pass, where the calculation is carried out starting from the initial node to the final node.

$$EF = ES + t,$$
$$ES = \max\{EF \text{ all direct predecessors}\},$$
$$ES_j = \max\left\{ES_i + t_{i,j}, i < j\right\}, \tag{1}$$

Backward Pass, where the calculation is carried out starting from the final node to the initial node.

$$LS = LF - t,$$
$$LF = \min\{LS \text{ all activities that immediately follow}\},$$
$$LF_i = \min\{LF_j - t_{i,j}, i < j\}, \tag{2}$$

where t is the duration or time of the activity.

If both calculations are completed, then slack time can be obtained. In general, slack time is given in the following equation:

$$\text{Slack time} = LF - ES. \tag{3}$$

Activities with a slack time value of 0 are usually called critical activities, and will later be connected with other critical activities to determine the critical path in the network..

2.2 Project Crashing

In certain situations, some or all of the existing activities can be accelerated to complete the project within the desired time limit. Heizer and Render [4] mention that the process which shortens the project duration with the lowest possible cost is called project crashing. There is a term cost slope which states the magnitude of the change in costs when an activity is accelerated or decelerated. Mathematically the cost slope perperiod can be formulated as follows:

$$\text{Cost slope} = \frac{C_c - C_n}{T_n - T_c}, \tag{4}$$

where C_c is the crash cost, C_n normal cost, T_n normal duration, and T_c is the crash duration.

2.3 Program Evaluation and Review Technique

In CPM, the assumption for all activities time is fixed and is known with certainty, whereas in PERT it is assumed that the duration of each activity has a range that follows a statistical distribution. Heizer and Render [4] state that the PERT is defined as a method that applies the determination of project time estimates based on situations that reflect uncertainty (probabilistic), using the distribution of opportunities based on three time estimations for each activity, namely realistic time m, optimistic time a, and pessimist time b, which will be used to get the expected time te with the following formula:

$$te = \frac{a + 4m + b}{6} \tag{5}$$

Meanwhile, to determine the percentage of project success, the standard deviation and variance of activities and projects are first calculated, which are mathematically formulated as follows:

$$S = \frac{b - a}{6} \tag{6}$$

$$\sigma := S^2 = \left(\frac{b - a}{6}\right)^2 \tag{7}$$

$$\sigma_p^2 \coloneqq \text{project variance} = \sum(\text{critical activity variance}) \tag{8}$$

$$\sigma_p := \sqrt{\text{project variance}} = \sqrt{\sigma_p^2}\,, \tag{9}$$

where S is the activity standard deviation, σ activity variance, σ_p^2 project variance, and σ_p is the project standard deviation.

Furthermore, a search for the relationship between the expected time te and the target duration Td can be performed which is expressed in the following formula:

$$z = \frac{Td - te}{\sigma_p} \tag{10}$$

where z is the number of possible target durations reached which later refers to the normal probability distribution.

3. RESULTS AND DISCUSSION

The authors discuss the analysis of project scheduling using the CPM and PERT methods with the help of Excel Solver software. The project used as a case study in this research is a pressure vessel manufacturing project, namely Project 595-PTS-PJR-001 RB BRONANG GAS SEPARATOR (20MBD-003) with a contract value of Rp. 618,000,000 and PT.Transalindo Eka Persada as project executor. The duration of completion of this project is 160 days. A sketch of the BRONANG GAS SEPARATOR pressure vessel manufacturing project can be seen in Figure 1.

The first step in analyzing project scheduling is to collect all data related to planning and scheduling. The data used in this study is in the form of a list of project activities, the time of each activity, the project implementation schedule, and the cost of each project activity. Next, prepare a network for the pressure vessel manufacturing project based on the data obtained for each of the CPM and PERT methods.

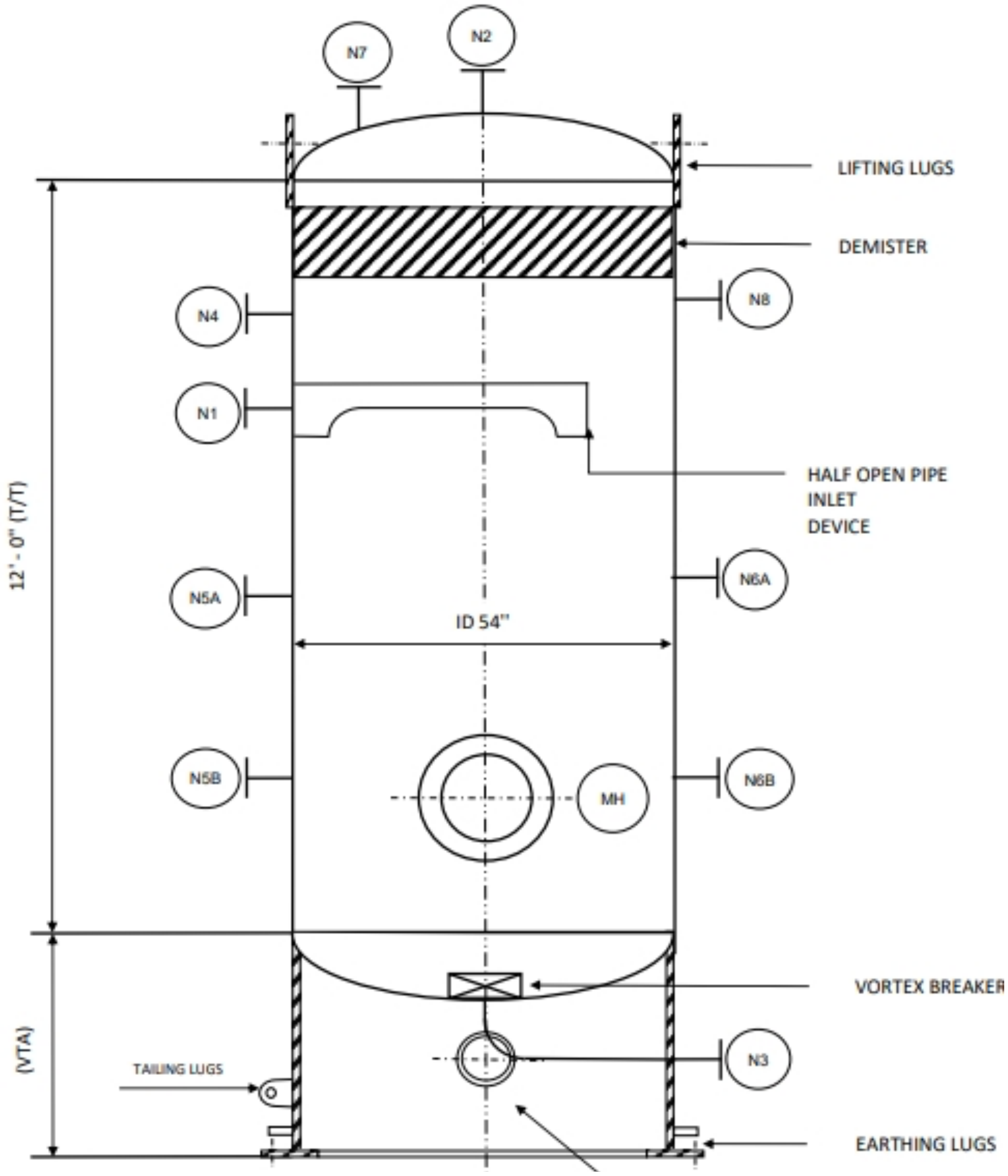
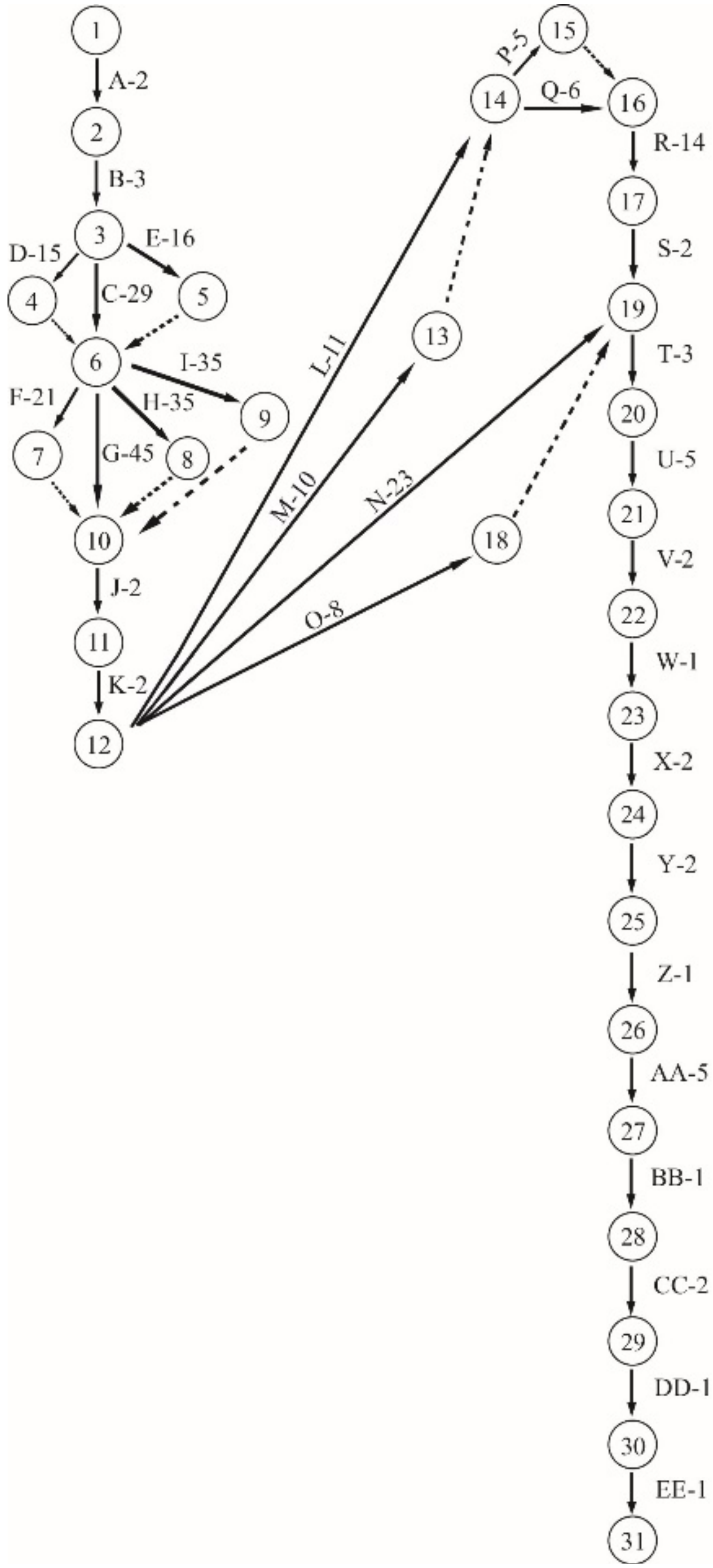


Figure 1. BRONANG GAS SEPARATOR pressure vessel sketch



1
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B-3
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D-15
E-16
C-29
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I-35
F-21
H-35
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G-45
8
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J-2
11
K-2
12
L-11
M-10
N-23
O-8
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18
P-5
15
14
Q-6
16
R-14
17
S-2
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T-3
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U-5
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W-1
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DD-1
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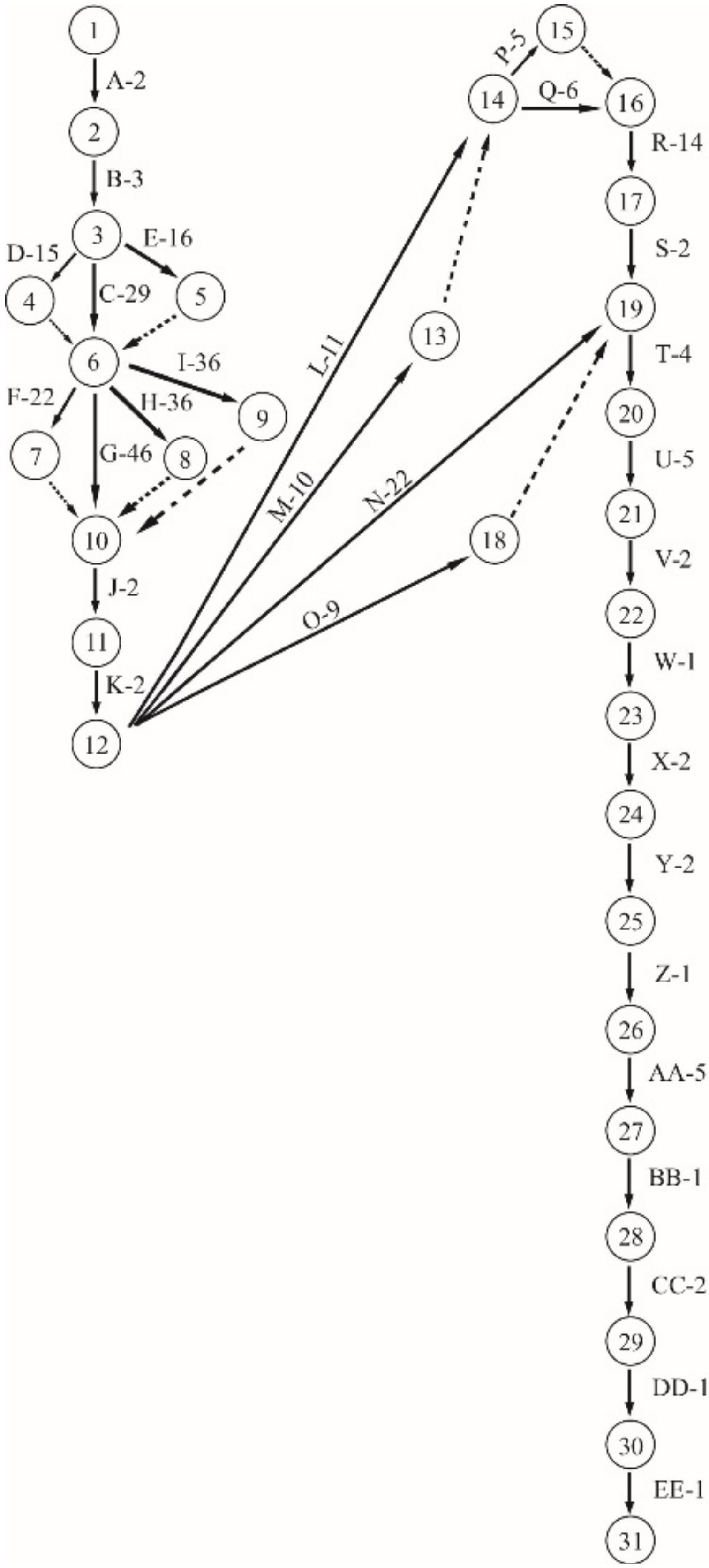
EE-1
31

(a)



(b)

Figure 2. (a) CPM network and (b) PERT network

In using the CPM and PERT methods in project scheduling analysis, it is necessary to find the critical path first. Determination of the critical path is done by a two-pass process. The initial time calculation is carried out by forward calculation starting from the first network node. Calculating ES activity at node 1 starts with $ES_1 = 0$. The next calculation can be seen as follows:

$$
\begin{aligned}
ES_2 &= \max\{ES_1 + t_{1,2}\},\\
&= \max\{0 + 2\},\\
ES_2 &= 2,\\
ES_3 &= \max\{ES_2 + t_{2,3}\},\\
&= \max\{2 + 3\},\\
ES_3 &= 5.
\end{aligned}
$$

Then if there are two or more arrows moving towards the same node, then the ES value taken is the largest ES value as happened at node 19.

$$
\begin{aligned}
ES_{19} &= \max\{ES_{12} + t_{12,19}, ES_{17} + t_{17,19}\},\\
&= \max\{83 + 23, 114 + 2\},\\
ES_{19} &= 116.
\end{aligned}
$$

the forward calculation process carried out above is continued until the last node, namely node 31.

After calculating the initial time using the forward calculation technique, then the completion time calculation is carried out using the backward calculation technique. Calculating the LF of activity at node 31 starts with $LF_{31} = 142$. The next calculation can be seen as follows:

$$
\begin{aligned}
LF_{30} &= \min\{LF_{31} - t_{30,31}\},\\
&= \min\{142 - 1\},\\
LF_{30} &= 141,\\
LF_{29} &= \min\{LF_{31} - t_{29,30}\},\\
&= \min\{141 - 1\},\\
LF_{29} &= 140.
\end{aligned}
$$

Then if there are two or more arrows moving towards the same node, then the LF value taken is the smallest LF value as happened at node 14.

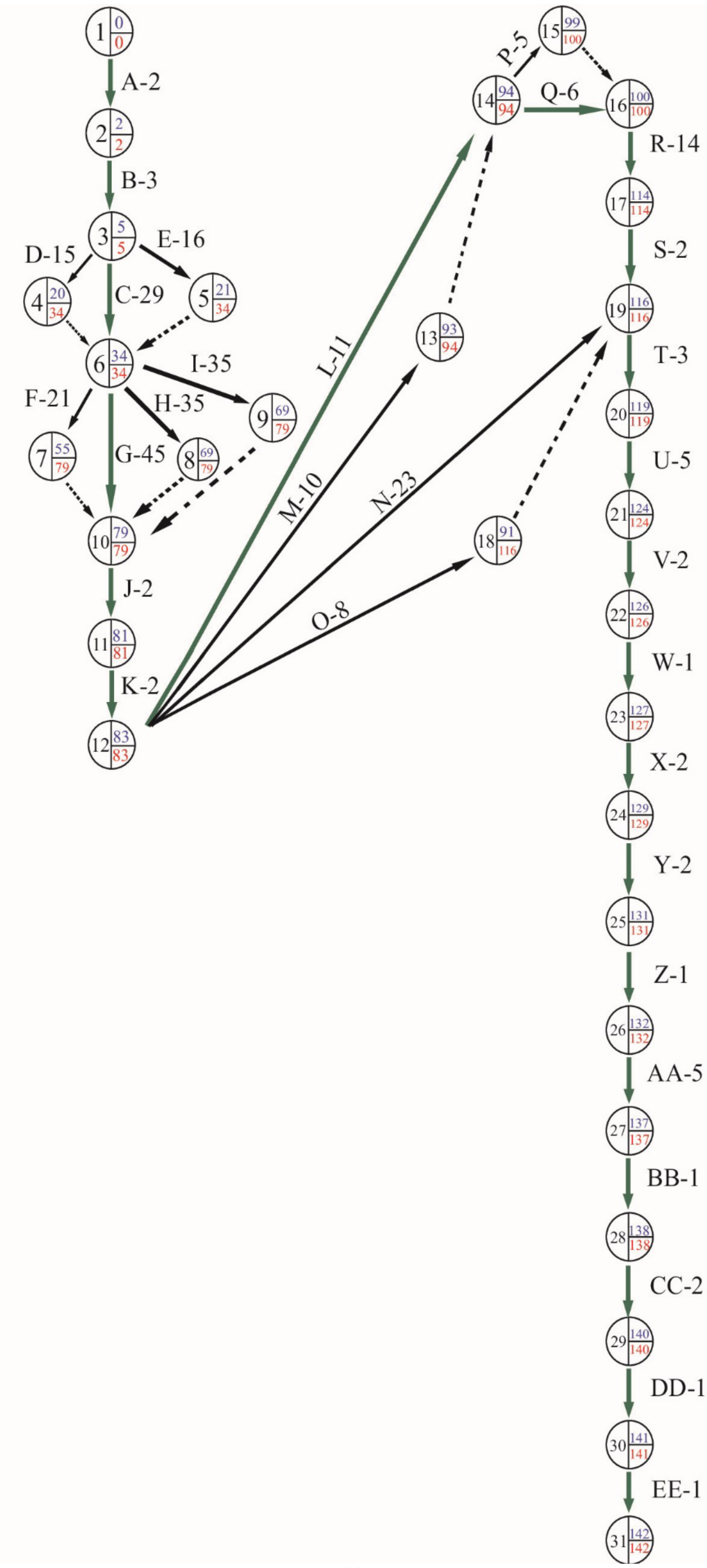
$$
\begin{aligned}
LF_{14} &= \min\{LF_{15} - t_{14,15}, LF_{16} - t_{14,16}\},\\
&= \min\{100 - 5, 100 - 6\},\\
LF_{14} &= 94.
\end{aligned}
$$

the calculation process is continued until the last node.

After performing forward and backward calculations, a slack time will be obtained which is obtained from the difference between LF and ES. The difference value of LF and ES which has a value of 0 is obtained at nodes 1, 2, 3, 6, 10, 11, 12, 14, 16, 17, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, and 31. The arrows connecting network nodes that have a slack time value of 0 are found in arrows A-B-C-G-J-K-L-Q-R-S-T-U-V-W-X-Y-Z-AA-BB-CC-DD-EE. The arrows are critical activities in the BRONANG GAS SEPARATOR pressure vessel manufacturing project. So, it can be obtained that the overall completion time for the BRONANG GAS SEPARATOR pressure vessel manufacturing project using the CPM method will take 142 days.

In calculations using the PERT method, the same calculation procedure is carried out as was done in the previous calculation of the CPM method. However, what distinguishes the PERT calculation here is the use of the duration value in the PERT method using the te duration value. After carrying out the calculation steps, it can be obtained that the overall project completion time using the PERT method will take 144 days. The critical path diagram for the CPM and PERT methods can be seen in Figure 3 as follows.

(a)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N-23
O-8
P-5
Q-6
R-14
S-2
T-3
U-5
V-2
W-1
X-2
Y-2
Z-1
AA-5
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DD-1
EE-1

(b)

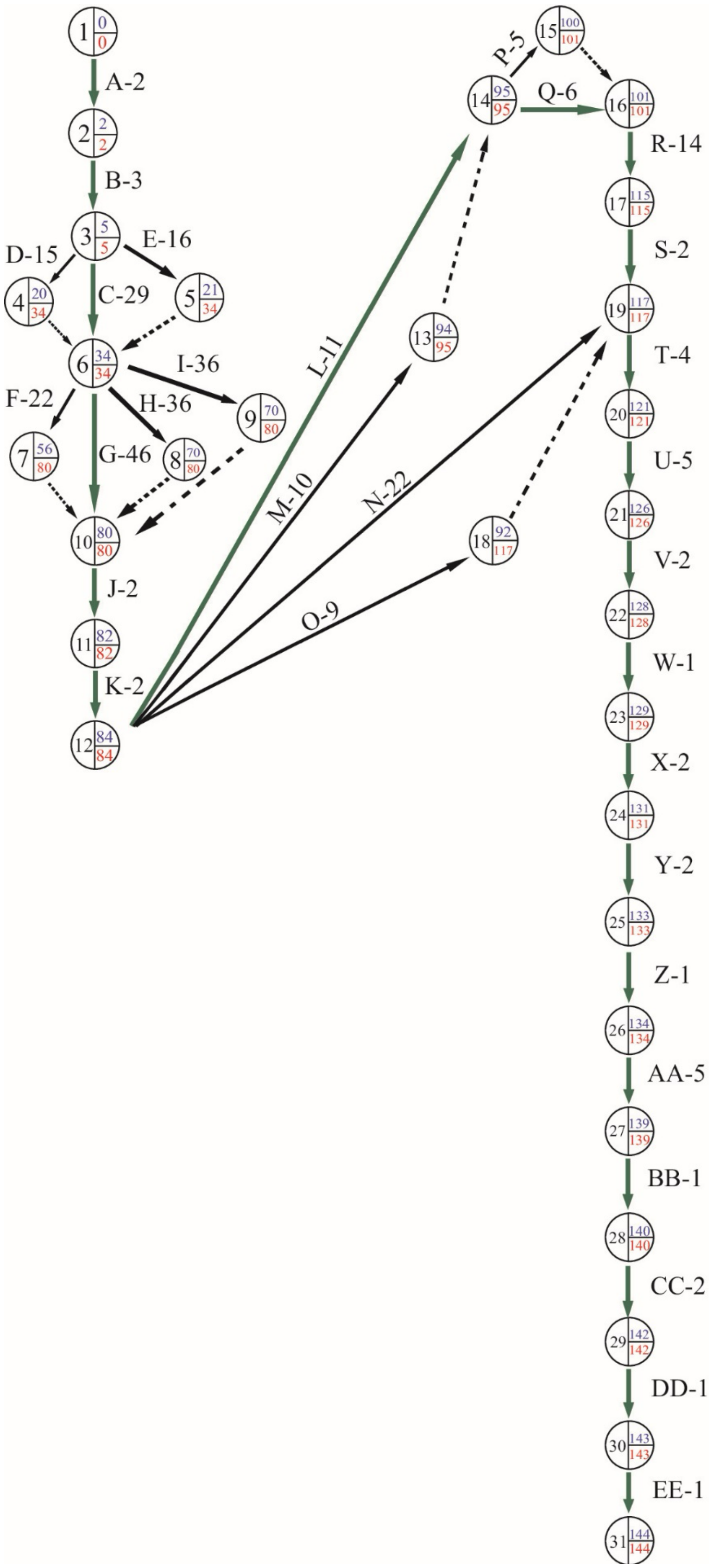

Figure 3. (a) CPM critical path and (b) PERT critical path

In calculations using the PERT method, standard deviation and variance are also calculated to determine the percentage of project success. Based on the variance value data for each activity, the project variance and project standard deviation are obtained as follows:

$$\sigma_p^2 = 0.11 + 0.03 + 0.25 + 1.36 + 0.00 + 0.03 + 0.44 + 0.11 + 0.11 + 0.03 + 0.44 + 0.25 + 0.00 + 0.03 + 0.00 + 0.00 + 0.00 + 0.25 + 0.03 + 0.03 + 0.00 + 0.03 = 3.53$$

$$\sigma_p = \sqrt{\sigma_p^2} = \sqrt{3.53} = 1.88$$

Furthermore, it can be seen that the probability of the project will be completed at the targeted or desired time. For example, the target duration value of Td is taken as 149 days, it can be calculated as follows:

$$z = \frac{Td - te}{\sigma_p} = \frac{149 - 144}{1.88} = 2.93$$

Referring to the normal distribution table, for z = 2.93, the probability that the pressure vessel manufacturing project will be completed within 149 days is 0.9983 or 99.83%.

In calculations using the CPM method, the overall duration of the pressure vessel manufacturing project takes 142 days. In some cases, the client may ask to accelerate the completion of the project for several reasons, so that it is necessary to accelerate the project as much as possible with the minimum possible cost. With the help of the Microsoft Excel Solver, the project duration was accelerated from 142 days to 132 days, in order to obtain crash duration and crash costs from activities that could be accelerated as can be seen in Table 1.

TABLE 1. List of crashing activities

Activity	Normal duration	Crash duration	New duration	Total cost
C	29	2	27	IDR 111,000
G	45	3	42	IDR 12,867,000
L	11	1	10	IDR 1,848,000
Q	6	1	5	IDR 1,848,000
R	14	1	13	IDR 3,695,000
T	3	1	2	IDR 1,848,000
U	5	1	4	IDR 1,848,000

Based on the results obtained from Table 1, the duration of completion of the BRONANG GAS SEPARATOR pressure vessel project can be accelerated to 132 days with a total crash cost or additional cost of IDR 24,065,000.

4. CONCLUSIONS

Based on the discussion above, it can be concluded that the total duration of completion for the BRONANG GAS SEPARATOR pressure vessel project using the critical path method is faster than the program evaluation and review technique method. The CPM method will be very suitable for projects that have been carried out repeatedly because the duration of the activity uses a single time estimate and it will be difficult to apply it to projects with new cases whose duration still often depends on many factors so that it is probabilistic.

However, in carrying out a project there is often a factor of uncertainty regarding the completion of project activities, so that the time for completion of activities varies and depends on many factors. This resulted in the need for the PERT method because it considers the time variability of project activities, even though in accurately setting the time required very in-depth experience from the project manager. The PERT method should also be used in optimizing project scheduling to see the probability of project completion within the desired deadline.

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